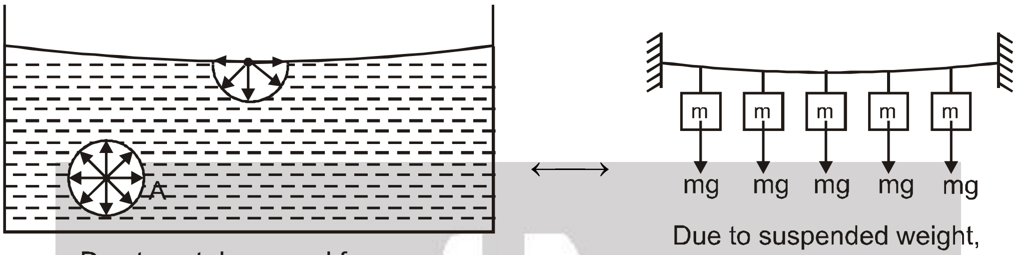
SURFACE TENSION

"Tension force generated in (applied by) the liquid surface is called surface tension force". In the fluid mechanics we have studied about the inner part of liquid, but in this chapter we will concentrate only on the surface of the liquid. The forces on the surface molecules are slightly different than the forces on inner molecules. Lets see how !

Explanation of surface tension on the basis of intermolecular forces :



Due to net downward force, a tension is generated in the surface

Due to suspended weight, tension is generated in rope

Actually surface tension is created due to cohesive forces, which is attractive force between the molecules of same substance.

Figure shows a container filled with a liquid. Consider a molecule 'A' which is inside the liquid. Equal cohesive force from all the direction acts on it. So net cohesive force on it is zero.

So cohesive force is meaningless for the liquid inside. That 's why we didn't used it in fluid mechanics.

Now lets consider a molecule 'B' on the surface. Water molecules are only below it, but there is no water molecule above it. So only the water molecules below it applies cohesive forces, and the resulting cohesive force is downwards.

Due to this downward force, a tension is generated in the surface, just like due to suspended weight, tension is generated in the rope.

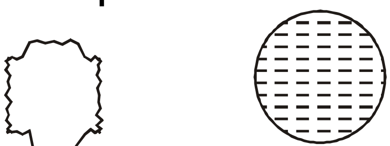
The tension generated in the surface is called surface tension force. Due to surface tension, the liquid surface behaves like a stretched membrane (rubber sheet) and try to minimize its area.

Explanation of surface tension on the basis of energy :

As we have seen, the molecule inside the liquid is attracted by the surrounding liquid molecules from all the directions. So it will has more negative energy (say –10). But the molecule on the surface is surrounded by liquid molecules only in lower half. So it will have less negative energy (say –5)

Less negative means more energy. So the molecules of surface have more energy than the molecule inside. For stability, the energy should be minimum possible. For minimum energy, the surface molecules should be minimum and hence surface area should be minimum. So the surface tries to minimize its area and due to this a tension is generated in the surface.

Some simple evidence of surface tension :

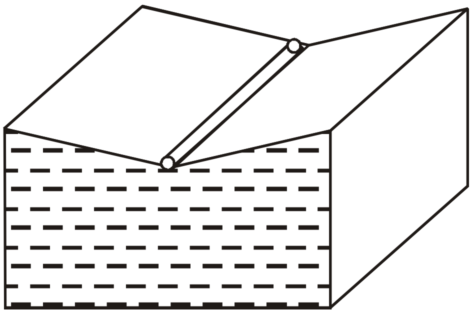


(i) Piece of stone

Piece of water (water drop)

A piece of stone can be of random shape because solids don't have surface tension. But a piece of water (water drop) is in spherical shape. Since there is tension in surface of water. So the water surface act like a tight membrane (tight bag). To minimize its surface area, the water drop takes spherical shape. For small drop gravitational pressure (ρgh) is negligible so the small drop is almost spherical. But in big drop gravitational pressure (ρgh) is considerable so the big drop has oval shape.

(ii) If we put a needle very slowly on the water surface, it will float on the surface as if it were put on a tight membrane. This also proves that there is a tension in the liquid surface due to which it act like a tight membrane.



(iii)

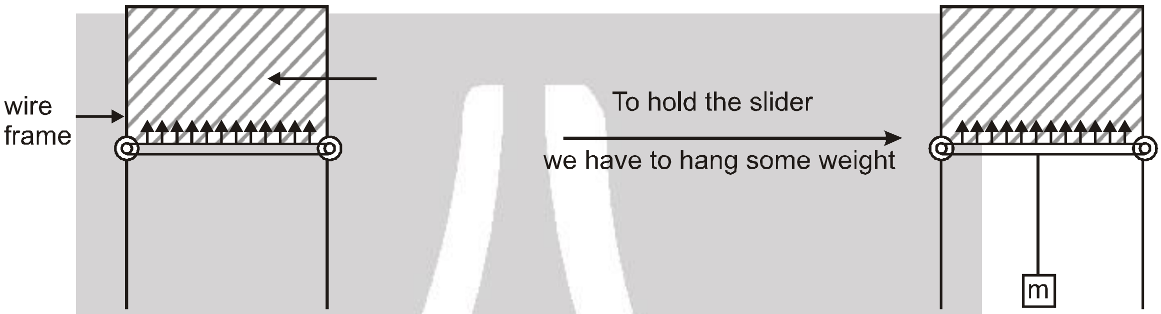
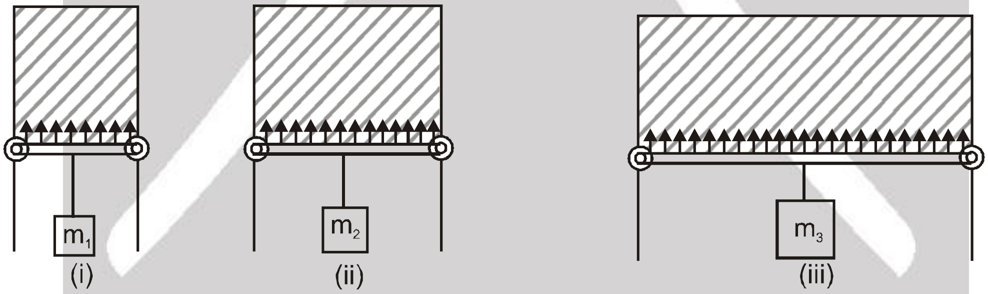
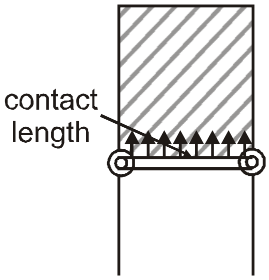


Figure shows a U shaped fixed wire frame, on which very light slider can slide. Dip the frame in soap solution and take it out. A thin film of soap solution is formed between the frame and slider, which is purely a surface. Now if we release the slider, it will move upwards, this shows that there is a tension in the liquid surface. The liquid surface applied tension force (pulling force) on the slider in contact, due to which the slider try to move upward. To keep the slider in equilibrium, we have to hang some weight. This is very close example. From this, we can also measure surface tension force.



Consider three cases (i), (ii) and (iii). In which case, the surface tension force on the slider is more? Practically it is observed that in case (i) surface tension force on slider is least, it is more in case (ii) and most in case (iii). In case (iii), we have to hang more weight to keep the slider in equilibrium. From this example it is clear that surface tension force depends on contact length which is greatest in case (iii)



Surface tension force (F) $\propto$ contact length (ℓ)

$$F = (T)\,\ell$$

Here T is a constant which is called surface tension constant. T depends on the properties of liquid and also on the medium which is on the other side of liquid.

- If we increase the temperature, surface tension constant (T) decreases.
- If we add highly soluble substances like NaCl, $ZnSO_4$ etc. then surface tension constant (T) increases.
- If we add sparingly soluble substances like soap, phenol, then surface tension (T) decreases.

Result :

Surface applies tension force (pulling force) on the other part of surface and also on any object (like slider) which is in contact.

Surface tension force

F = (T) (ℓ) where ℓ = contact length = length of Boundary line between the two surfaces

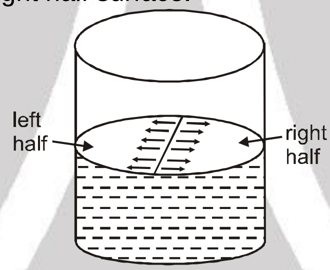
also T = so the definition of surface tension (T) can be written as

T = F/ℓ

The surface tension of a liquid can be measured as the force per unit length on an imaginary line drawn on the liquid surface, which acts perpendicular to the line on its either side at every point and tangentially to the liquid surface.

Solved Example

Example 1. Figure shows the container of radius R filled with water. Consider an imaginary diametric line dividing the surface in two parts: Left half and right half. Find surface tension force between the left half surface and the right half surface.

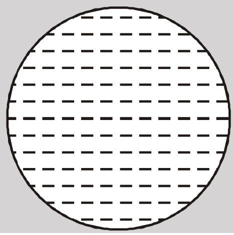


Solution : Both left half and right half surface will pull each other with a force F = (T) (ℓ)

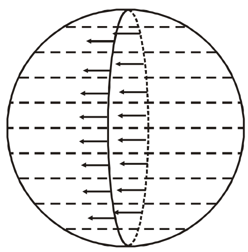
where ℓ is the length of boundary lines between the two surfaces which is equal to 2R

So F = (T) (2R)

Example 2. Consider a water drop of radius R. Find surface tension force between the left half surface and right half surface ?



Solution : Surface tension force

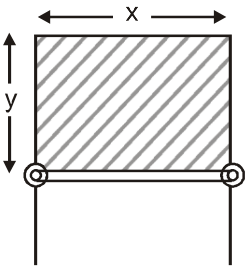


F = (T) (ℓ)

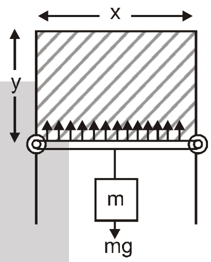
here ℓ = length of boundary line between left half and right half surface = $2\pi R$

So F = (T)($2\pi R$)

Example 3. Between a frame and a light slider, a thin film of soap solution is made. Whose length is x and width is y. Find surface tension force on the slider. To keep the slider in equilibrium, how much weight should be suspended ?



Solution : The surface will act like a tight membrane and pull the slider with a force F = (T) (ℓ). Since this a film, it will have two surfaces: the front surface and the back surface. On the front surface, contact length is x, and also on the back surface contact length is x. So total contact length will be ℓ = x + x = 2x



So surface tension force on slider.

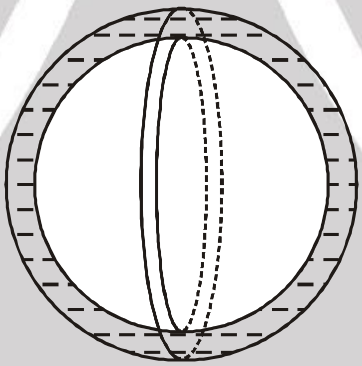
F = (T)(2x)

For equilibrium, this force will be balanced by weight of suspended block.

(T)(2x) = mg

$$m = \frac{2Tx}{g}$$

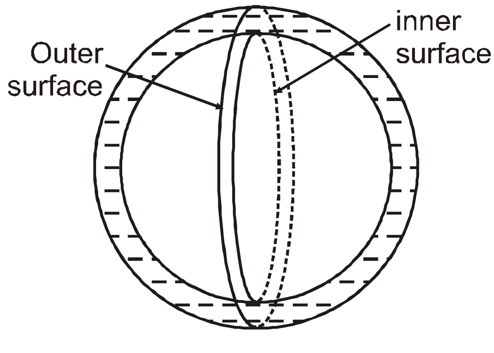
Example 4. Consider a bubble of soap solution. Find the surface tension force between the left half surface and right half surface



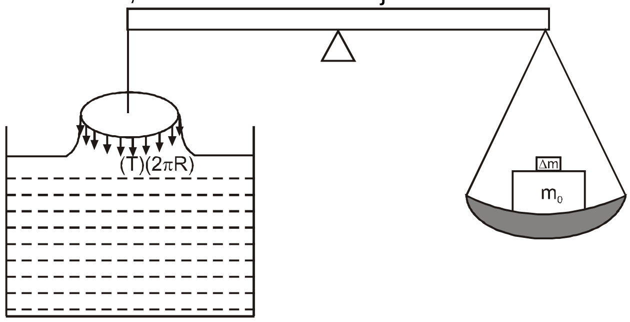
Solution : The bubble also have two surfaces: the inner surface and the outer surface. And in the small thickness between them, there is some liquid. So the surface tension force will be applied by inner surface as well as the outer surface [T(2πR)].

So total surface tension force between left half and right half surface is

F = (T)(2πR) + (T)(2πR) = T(4πR)



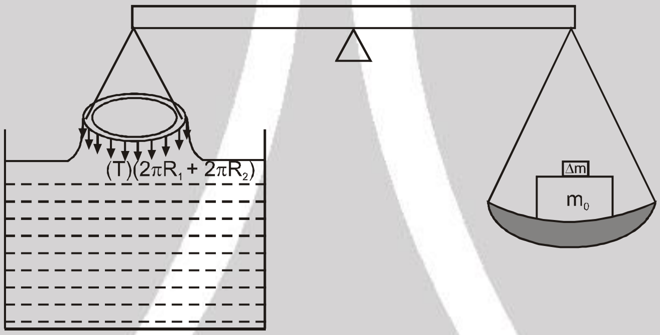
Example 5. A thin disc of radius R, just touching the liquid surface, forms one arm of a balance. The plate is balanced by some weight on the other side of the balance. How much extra weight should be added on the other side, so that the disc can just come out of water ?



Solution: Surface tension force on the disc is (T)(2πR)

For balance $(T)(2\pi R) = (\Delta m)g \Rightarrow \Delta m = \dfrac{(T)(2\pi R)}{g}$

Example 6. In the previous question, in place of disc a ring is used whose inner radius is R_1 and outer radius is R_2. Now how much extra weight should be added on the other side, so that the ring can just come out of water ?

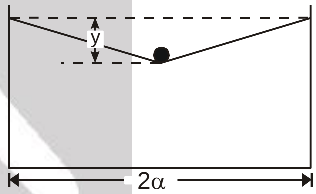


Solution : Surface tension force on the disc is $(T)\, 2\pi\, (R_1 + R_2)$

For balance $(T)\, 2\pi(R_1 + R_2) = (\Delta m)g \Rightarrow \Delta m = \dfrac{(T)2\pi(R_1 + R_2)}{g}$

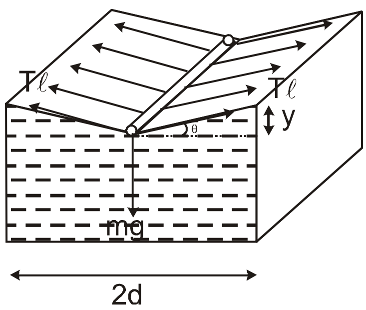
Example 7. **(Only for JEE Advanced)**

A long thin straight uniform wire of negligible radius is supported on the surface of a liquid. The width of the container is 2d and the wire is kept at its centre, parallel to its length (as shown in figure). The surface of the liquid is depressed by a vertical distance y(y << d) at the centre as shown in figure. If the wire has mass λ per unit length, what is the surface tension of the liquid? Ignore end effects.



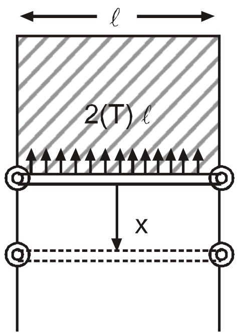
Solution : Balancing upward an downward forces $2(T\ell \sin\theta) = (\lambda\ell)g$

as angle is very small $\sin \approx \tan\theta = \dfrac{y}{d}$



$2(T\ell)\dfrac{y}{d} = (\lambda\ell)g \quad T = \dfrac{\lambda g d}{2y}$

Surface energy :



Potential energy stored due to surface tension force is called surface energy. To understand this, suppose a thin film of soap solution is formed between the fix frame and the slider. Both front and the back surface will pull the slider with a force of $F = 2(T\ell)$

Now we move the slider forward by a distance x.

During this :

Work done by surface tension force = $-(2T\ell)(x)$

(As surface tension force is opposite of displacement)

$\Rightarrow$ Work done against surface tension force = $+(2T\ell)x$

$\Rightarrow$ Increase in surface potential energy = $+(2T\ell)x$

where $2\ell x$ = increasing surface area (increase in front area = ℓx, increase in back area = ℓx)

$\Rightarrow$ Increase in surface potential energy $\Delta U = (T)(\Delta A) = (T)$ (increase in surface area)

or generally, we can say that

Surface energy $U = (T)(A) = (T)$ (surface area)

also $T = \frac{U}{A}$ and previously we have seen that $T = \frac{F}{\ell}$

So Surface tension is surface energy per unit surface area

Surface tension is also tension force generated on the surface per unit length.

Solved Example

Example 8. 1000 small water drops, each of radius r, combine and form a big drop. In this process, find decrease in surface energy.

Solution : Suppose radius of big drop is R. During this process, mass will be conserved, so volume will also be conserved.

$(\text{Volume})_{\text{initial}} = (\text{Volume})_{\text{final}}$; $\left(\frac{4}{3}\pi r^3\right)\times 1000 = \left(\frac{4}{3}\pi R^3\right)$ $\Rightarrow$ $R = 10r$

loss in surface energy $\Delta U_{loss} = T\Delta A_{loss} = T(\,(4\pi r^2)\times 1000 - 4\pi(10r)^2\,)$

$\Rightarrow$ $\Delta U_{loss} = (T)\,(900\times 4\pi r^2)$

this energy loss will be converted into heat. So increase in temperature of the drop can be found from

$T(900\times 4\pi r^2) = ms\Delta T$, From this get the increase in temperature ΔT.

Example 9. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given be $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ where T is the surface tension of water and J is the mechanical equivalent of heat. Here r, R and T are in CGS system.

Solution : suppose n small water drop combine and form a big drop. During this process so volume will also be conserved

$(\text{Volume})_{\text{initial}} = (\text{Volume})_{\text{final}}$

$$\left(\frac{4}{3}\pi r^3\right)\times n = \frac{4}{3}\pi R^3 \quad\Rightarrow\quad n = \frac{R^3}{r^3}$$

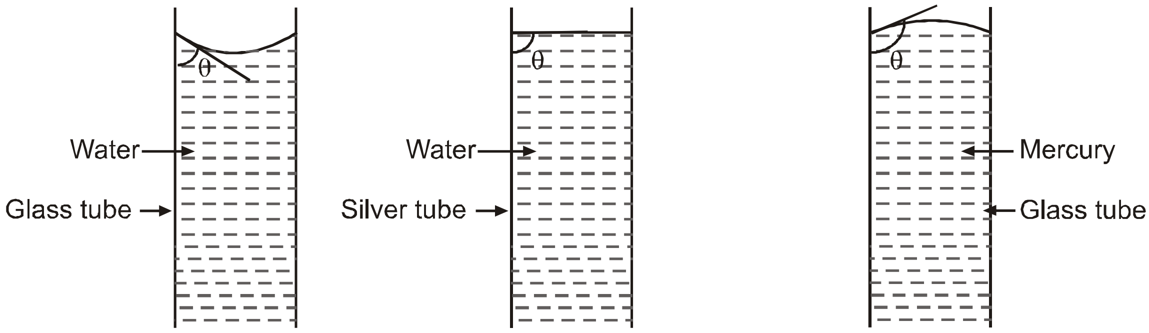
Loss in surface energy $\Delta U_{loss} = T\Delta A_{loss} = T(4\pi r^3\times n - 4\pi R^2)$

Put $n = \frac{R^3}{r^3}$ get $\Delta U_{loss} = T\left(4\pi r^2\times\frac{R^3}{r^3} - 4\pi R^2\right)$; $\Delta U_{loss} = T\,4\pi R^3\left(\frac{1}{r}-\frac{1}{R}\right)$

$T(4\pi R^3)\left(\frac{1}{r}-\frac{1}{R}\right) = ms\,\Delta\theta$ when $m = \rho(\text{vol})$

$T(4\pi R^3)\left(\frac{1}{r}-\frac{1}{R}\right) = (1\ \text{gm/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1\ \text{cal/gm}\ ^oC)\Delta T$ get $\Delta\theta = \frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$

SHAPE OF LIQUID SURFACE :



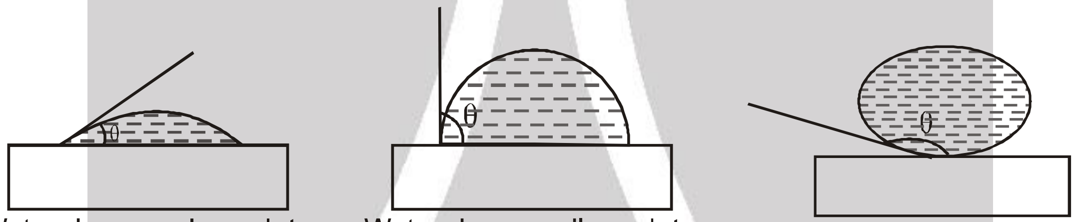
(i) (ii) (iii)

If we fill water in glass tube, the surface becomes concave in shape, if water is filled in silver tube, the surface becomes horizontal and if Hg is filled in glass tube, the surface becomes convex.

Shape of liquid surface is called meniscus. At point of contact, angle between the tangent to the liquid surface and solid surface submerged in liquid is called angle of contact (θ).

In figure (i) angle of contact is acute.

In figure (ii), angle of contact is 90° and in figure (iii) angle of contact is obtuse. Angle of contact can also be observed when a liquid drop is put on a plate as shown below :



Water drop on glass plate $\theta < 90°$ | Water drop on silver plate $\theta = 90°$ | Hg drop on glass plate $\theta > 90°$

The shape of liquid surface depends on cohesive and adhesive forces.

Cohesive force : The force of attraction between the molecules of the same substance is called cohesive force. The cohesive forces is effective if distance between molecules is less than 10^{-9} m. If distance between molecule is greater than 10^{-9} m then cohesive force is negligible. The sphere drawn around a particular molecule as centre and range of cohesive forces (10^{-9} m) as radius is called **sphere of influence (sphere of molecular activity)**. The centre of molecule is attracted by only the molecules lying inside the sphere of influence.

Example : cohesive force between water molecules.

On the corner molecule (see the figure (i) (a) bellow), all the neighbouring water molecules will apply cohesive force, so net cohesive force (F_c) on it can be assumed to be centered at 45° angle with vertical.

Other examples of Cohesive force :

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohe sive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

Adhesive force : The force of attraction between different substances is called adhesive force.

Example Adhesive force between water and glass tube.

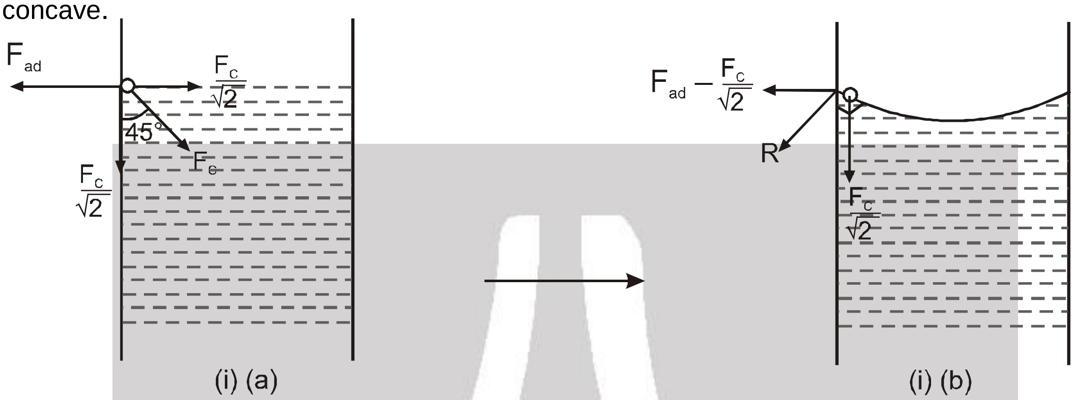
On the corner molecule, adhesive force will be towards the glass wall as shown in figure (i) (a) bellow.

Other examples of adhesive force :

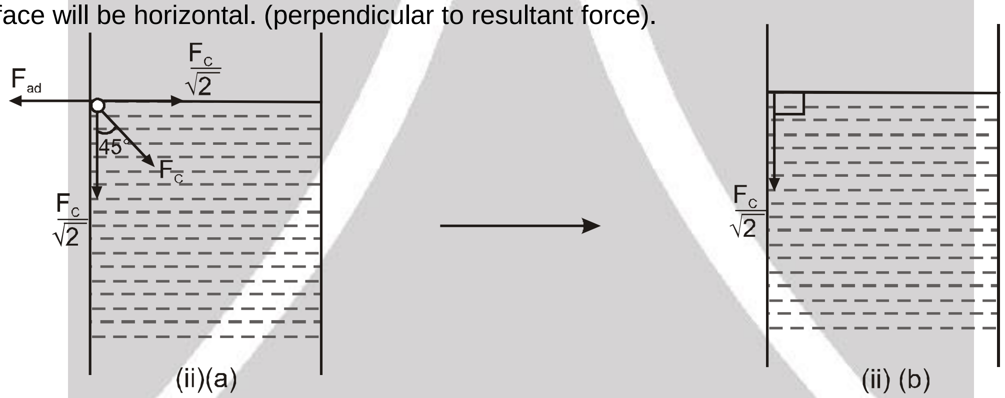
Examples.

(i) Adhesive force enables us to write on the black board with a chalk.
(ii) Adhesive force helps us to write on the paper with ink.
(iii) Large force of adhesion between cement and bricks helps us in construction work.
(iv) Due to force of adhesive, water wets the glass plate.
(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

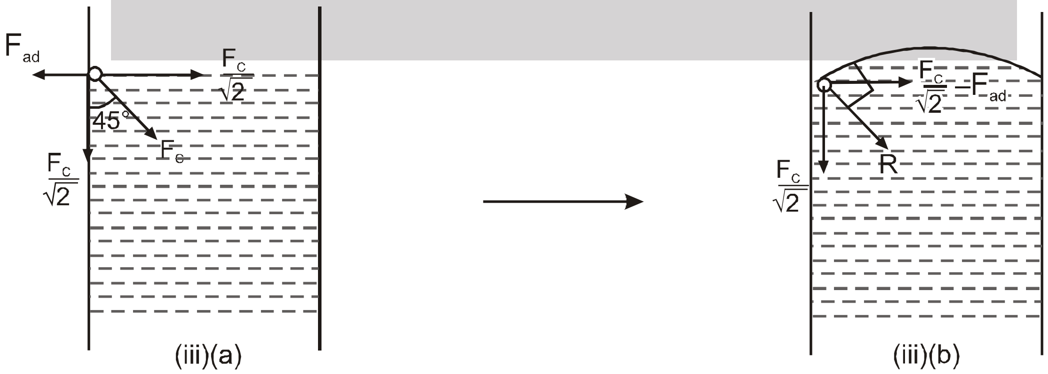
Case- I : If water is filled in a glass tube, $F_{ad} > \frac{F_c}{\sqrt{2}}$ then the resultant force will be as shown in (i) (b). As the water surface always adjusts itself perpendicular to the resultant force. So the surface will be concave.



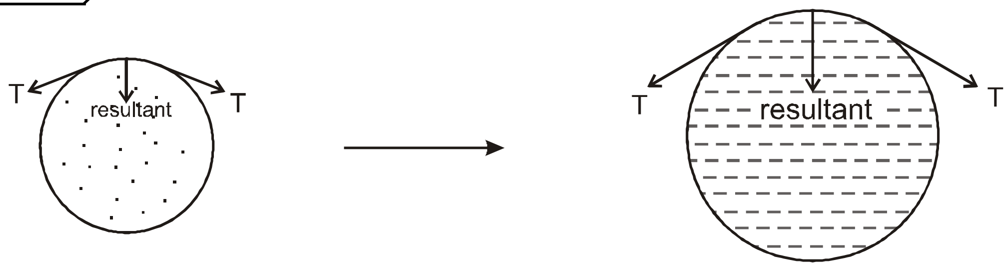
Case -II : If water is filled in silver tube, $F_{ad} = \frac{F_c}{\sqrt{2}}$ so resultant will be vertically downwards. So liquid surface will be horizontal. (perpendicular to resultant force).



Case -III : If Hg is filled in glass tube, $F_{ad} < \frac{F_c}{\sqrt{2}}$ so resultant force will be as shown in (iii) (b). As the surface adjusts itself perpendicular to the resultant force so surface will be convex.



PRESSURE EXCESS INSIDE A LIQUID DROP :

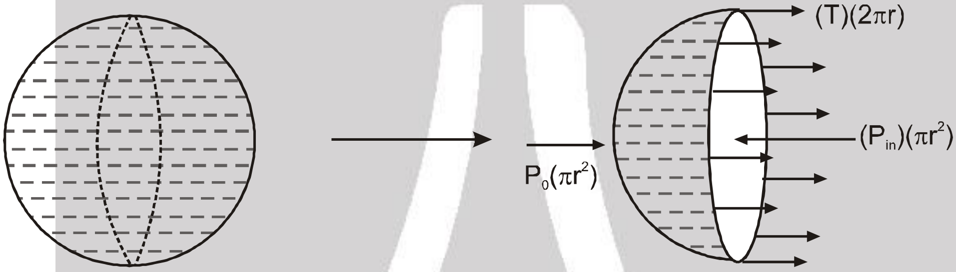


Due to stretched rubber, the air inside gets compressed. So pressure of air inside will be greater than pressure of air outside

The water surface also acts like a stretched rubber. So due to tension in the surface the water inside get compressed

So the pressure of water inside will be greater than the outside atmospheric pressure. This extra pressure is called pressure excess.

To find pressure excess, make free body diagram of the half part. The forces on this hemisphere are :



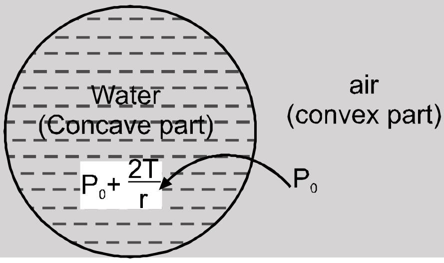
(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$

(ii) Pushing force due to atmospheric pressure will be (P_0) × (facing area) = $P_0(\pi r^2)$

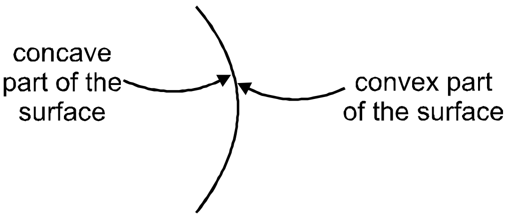
(iii) Surface tension force on left half surface due to right half surface will be (T)(2πr)

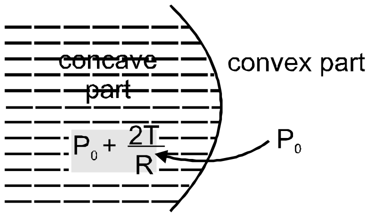
Applying force balance : $(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r)$

$\Rightarrow$ $P_{in} = P_0 + \frac{2T}{r}$, here $\frac{2T}{r}$ is called pressure excess. So pressure inside the drop will be greater than pressure outside the drop by $\frac{2T}{r}$



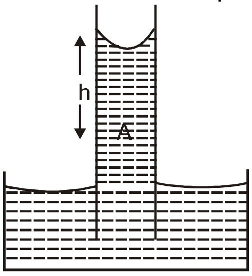
Generally we can say that pressure at concave part will be greater than pressure at convex part by $\frac{2T}{r}$ where r is radius of curvature of the surface between them.



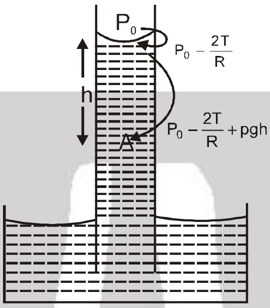


Solved Example

Example 10. Water is filled in a capillary tube of radius R. If the surface of water is hemispherical ($\theta = 0$), then find pressure at a point 'A' which is at h depth below the surface.

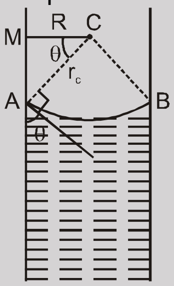


Solution :



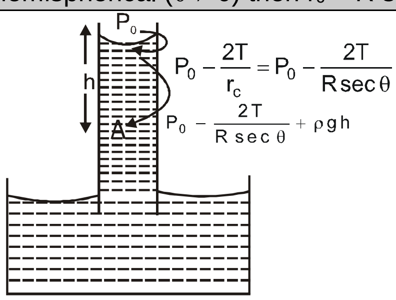
Water is on convex part. So pressure of water just below the surface will be less by $\frac{2T}{R}$. So pressure at point A is $P_0 - \frac{2T}{R} + \rho gh$. Here surface of water was hemispherical (contact angle $\theta = 0$) so radius of curvature of the surface = radius of the tube = R.

Example 11. In the previous question, suppose contact angle is not zero, but it is θ (the surface not hemispherical) now find pressure at point 'A'



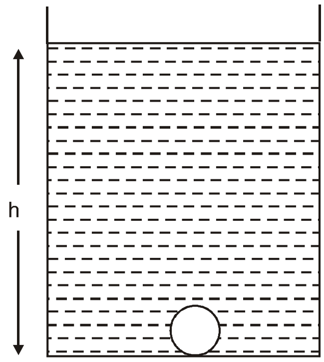
Solution : Draw normal (radial lines) at point A and B of periphery. The point (C) where radial lines meet is called centre of curvature. If contact angle is θ, from Δ ACM, $r_C = R\sec\theta$
So radius of curvature of the surface $r_C = R\sec\theta$.

Point to remember :
If the liquid surface is hemispherical ($\theta = 0$) then $r_c = R$
If liquid surface is not hemispherical ($\theta \neq 0$) then $r_c = R\sec\theta$

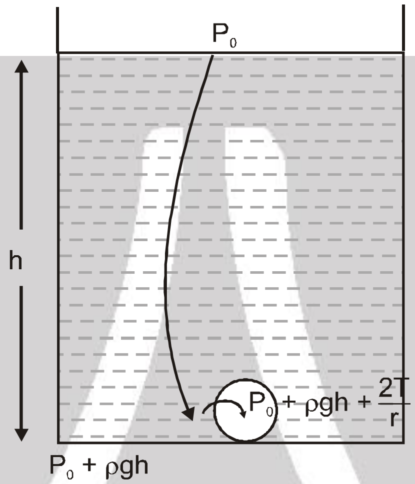


So pressure at A is $P_0 - \frac{2T}{R\sec\theta} + \rho gh$

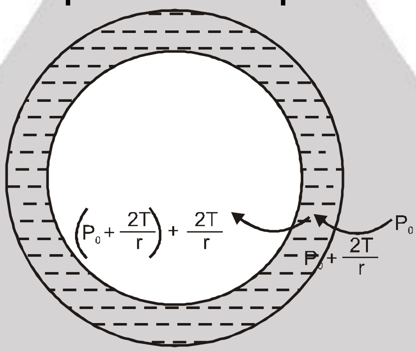
Example 12. A small air bubble (cavity of air) of radius r is at depth 'h' .Find the pressure inside the bubble.



Solution : Out of water and air (inside the bubble) air is on concave part.



Pressure excess inside a liquid bubble kept in air :-



So pressure inside the liquid bubble = $P_0 + \frac{4T}{r}$

So pressure excess inside the liquid bubble = $\frac{4T}{r}$

Alternative method :

Draw free body diagram of half part of bubble. The force on this hemisphere are :

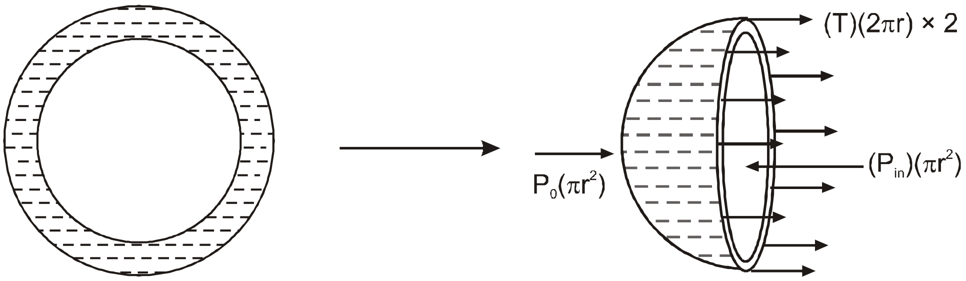
(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$

(ii) Pushing force due to atmospheric pressure will be (P_0) × (facing area) = $P_0(\pi r^2)$

(iii) Surface tension force on both inner and outer surface will be $(T)(2\pi r) \times 2$

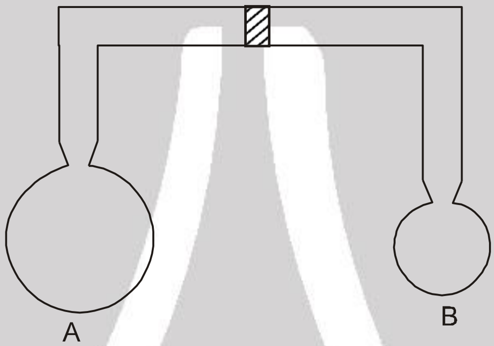
Applying force balance :

$(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r) \times 2 \Rightarrow \quad P_{in} = P_0 + \frac{4T}{r}$, So pressure excess inside a liquid bubble = $\frac{4T}{r}$

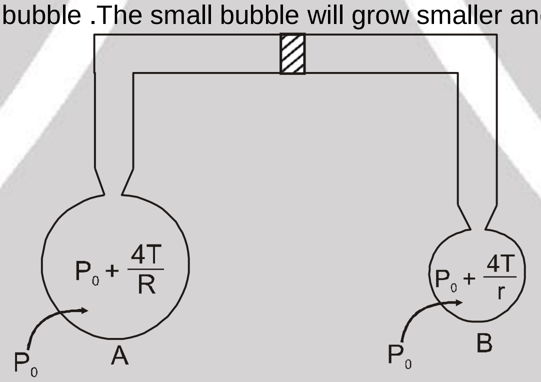


Solved Example

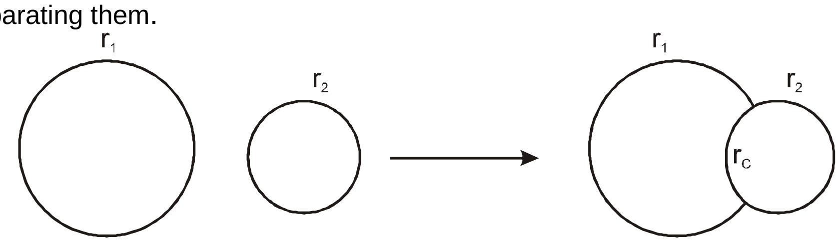
Example 13. Two soap bubbles are formed on the ends of the tube as shown. If valve is opened, in which direction will the air flow ?



Solution : Radius of curvature of smaller bubble 'B' is less so pressure inside the smaller bubble will be more $\left(P_0 + \frac{4T}{r}\right)$. Air will flow from high pressure to low pressure, so it will flow from smaller bubble to bigger bubble .The small bubble will grow smaller and the big bubble will grow bigger.

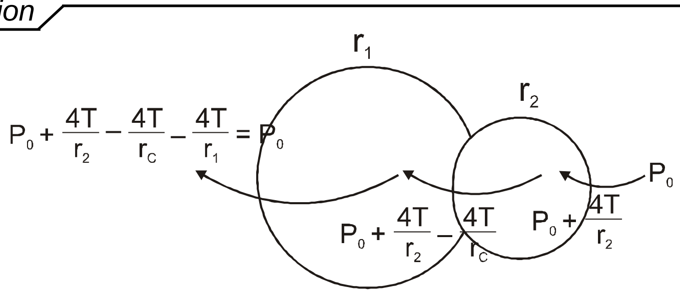


Example 14. Two soap bubbles of radius r_1 and r_2 combine .Find radius of curvature of the common surface separating them.



Solution : $P_0 + \frac{4T}{r_2} - \frac{4T}{r_c} - \frac{4T}{r_1} = P_0$

$$\frac{1}{r_c} = \frac{1}{r_2} - \frac{1}{r_1}$$



$$r_c = \frac{r_1 \ r_2}{r_1 - r_2}$$

Example 15. **(Only for JEE Advance)**

A soap bubble of radius r and surface tension constant T is given a charge, so that its surface charge density is σ. Due to charge, the radius of the soap bubble becomes double then find 'σ'. (atmospheric pressure = P_0)

Solution : Initial pressure inside the bubble $P_i = P_0 + \frac{4T}{r}$

Now a uniform surface charge in given to the bubble

The surface tension is a pulling force, which increases pressure inside the bubble (by $\frac{4T}{r}$)

But the charges given to the surface will repel each other. So due to the charge given, pressure inside the bubble will decrease (by $\frac{\sigma^2}{2\varepsilon_0}$)

So, final pressure inside the bubble $P_f = P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}$

As the temperature of the gas inside the bubble if constant so, $P_iV_i = P_fV_f$

$$\left(P_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}\right)\left(\frac{4}{3}\pi r_f^3\right)$$

Here Put $r_f = 2r$ So, get $\sigma = \sqrt{\left(7P_0 - \frac{12T}{r}\right) 2\varepsilon_0}$.

Example 16. **(Only for JEE Advance)**

A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Solution : The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1 \rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is $= (P + h_2 \rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyle's Law ; $P_1V_1 = P_2V_2$ where $V_1 = \frac{4}{3}\pi r_1^3$, and $V_2 = \frac{4}{3}\pi r_2^3$

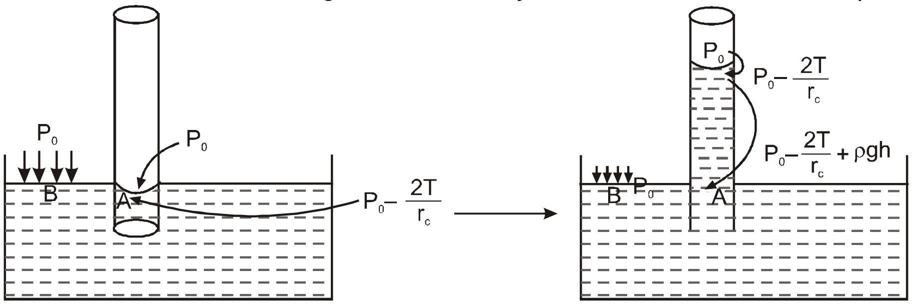
Hence we get $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\pi r_2^3$

or, $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right] r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right] r_2^3$

Given that : h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 mm = 0.0126 cm, T = 567 dyne/cm, P = 76 cm of mercury. Substituting all the values, we get

h_2 = 9.48 cm.

CAPILLARY ACTION : A glass tube of very small diameter is called capillary



If we dip the capillary tube in water, due to the concave surface, pressure just below the surface becomes $P_0 - \frac{2T}{r_c}$, while on the other points at the same horizontal level, pressure is P_0. Due to this less pressure water level in the tube rises up, till pressure becomes equal at the same horizontal level (At point A and B)

$$P_0 - \frac{2T}{r_c} + \rho gh = P_0 \Rightarrow h = \frac{2T}{\rho g r_c}$$

where r_c = radius of curvature of the water surface. If the water surface is hemispherical ($\theta = 0$), then $r_c = R$ but if water surface is not hemispherical ($\theta \neq 0$), then $r_c = R\sec\theta \Rightarrow h = \frac{2T\cos\theta}{\rho g R}$

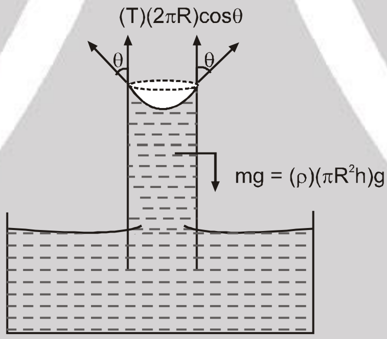
From this formula, we can say that

If $\theta < 90°$ then h = ⊕ ve, so water in capillary will rise up (Ex. water in glass tube).

If $\theta = 0$ so h = 0, so water in the capillary will not rise. (Ex. water in silver tube).

If $\theta > 90°$, h = –ve, so liquid in capillary will go down. (Ex. mercury in glass tube).

Deriving capillary rise from force balance :



As we dip the capillary in water, the surface pulls the capillary walls in downwards direction, so the capillary walls pulls the surface in upward direction as shown in figure, due to which water will rise up till the forces get balanced.

Lets draw free body diagram of the water raised up. Forces on it are :

(i) The surface pulls the capillary in downward direction, so as a reaction, the capillary pulls the surface in upward direction. Their horizontal components will be cancelled out and their vertical components will be added up. So net surface tension force will be vertically upwards and will be $(T)(2\pi R)\cos\theta$.

(ii) The weight of raised water; we can neglect the weight of meniscus. So the weight of raised water = $(\rho)(\pi R^2 h)g$

For equilibrium, forces should be balanced.

$$(T)(2\pi R)\cos\theta = (\rho)(\pi R^2 h)g \quad \Rightarrow \quad h = \frac{2T}{\rho g R}\cos\theta$$

From this equation we can say that $h \propto 1/R$. So if the capillary is thin, water will raise to more height.

If pure water is inside a glass tube, then $\theta \to 0$ so $h = \dfrac{2T}{\rho g R}$

Although in the previous derivation the volume of meniscus is negligeable, but if we have to consider the volume of meniscus then the volume of water raised will be $\pi r^2(h + r) - \dfrac{2}{3}\pi r^3$ so applying force balance

$(T)(2\pi R)\cos\theta = (\rho)(\pi r^2 (h + r) - \dfrac{2}{3}\pi r^3)g$ solving $\left(h + \dfrac{r}{3}\right) = \dfrac{2T}{\rho g R}\cos\theta$

Practical Applications of Capillarity

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tip of nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries must be. broken up. This is done by ploughing and leveling the fields
8. Bricks are porous and behave like capillaries.

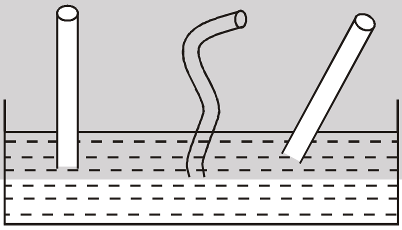
Solved Example

Example 17. A capillary of internal radius 4 mm, is dipped in water. To how much height, will the water rise in the capillary. ($T_{water} = 70 \times 10^{-3}$ N/m, g = 10 m/sec^2, ρ_{water} 10^3 kg/m^3, contact angle $\theta \to 0$)

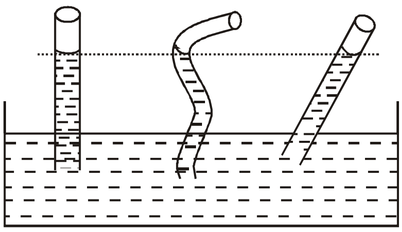
Solution : Capillary rise

$h = \dfrac{2T}{\rho g R}\cos\theta = \dfrac{2 \times 70 \times 10^{-3}}{10^3 \times 10 \times 4 \times 10^{-3}}$ .....(1) h = 3.5 mm

Example 18. If all the glass capillaries have same internal radius, then in which of the capillary, water will rise to move height ?



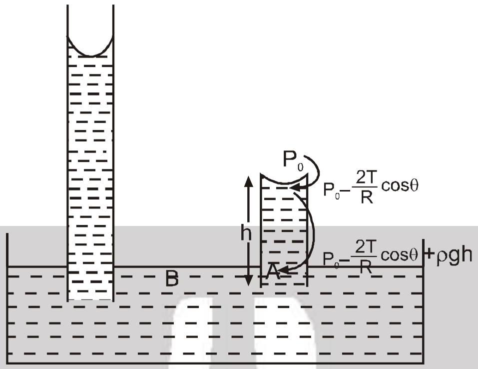
Solution : The height of water in the capillary $\left(h = \dfrac{2T}{\rho g r}\cos\theta\right)$ doesn't depend on shape of the capillary. So water will raise to same height in all the tubes. (However the length of water column in the tubes can be different)



If capillary tube of insufficient length is used :

Suppose a thin capillary tube of radius 0.35 mm is dipped in water. $T_{water} = 70 \times 10^{-3}$ N/m, $\theta \to 0$.
In this case water will rise up to a height

$$h = \frac{2T}{\rho g R}\cos\theta = \frac{2 \times 70 \times 10^{-3}}{10^3 \times 10 \times 0.35 \times 10^{-3}} = 4\text{cm}$$



Now suppose we use shorter capillary of same radius, but its length is only 2 cm. It is slightly dipped in the water.
To balance the pressure, water level will rise up in the capillary, it will reach upto the upper end of the tube, and now the contact angle will change till the pressure at same horizontal level is balanced. Balancing pressure at point A (inside the capillary) and point B (outside)

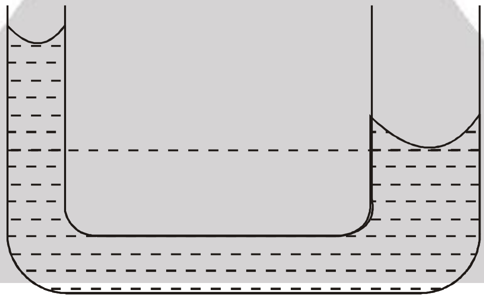
$$P_0 - \frac{2T}{R}\cos\theta + \rho g h = P_0 \quad \Rightarrow \quad h = \frac{2T}{\rho g R}\cos\theta$$

$$2 \times 10^{-2} = \frac{2 \times 70 \times 10^{-3}}{10^3 \times 10 \times 0.35 \times 10^{-3}}\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

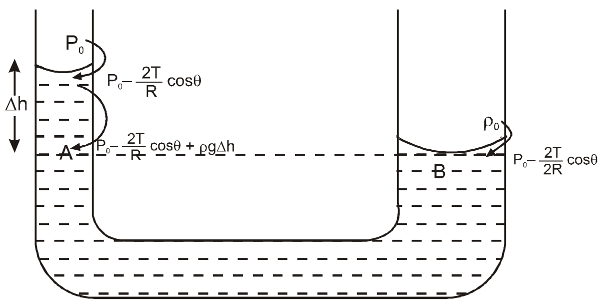
So water level will reach to the topmost point of the capillary (= 2cm) and now contact angle will change to 60°. Water will not overflow out of upper end in the form of fountain.

Solved Example

Example 19. In the U-tube, radius of one arm is R and the other arm is 2R. Find the difference in water level if contact angle is $\theta = 60°$ and surface tension of water is T.



Solution :

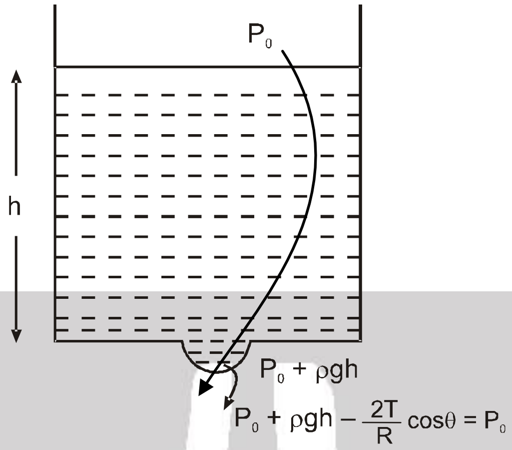


Balancing pressure at points A and B situated in same horizontal level.

$$P_0 - \frac{2T}{R}\cos\theta + \rho g\Delta h = P_0 - \frac{2T}{2R}\cos\theta \quad \text{here } \theta = 60°, \text{ solving we get } \Delta h = \frac{T}{2\rho g R}$$

Example 20. There is a small hole of diameter 0.1 mm at the bottom of a large container. To what minimum height we can fill water in it, so that water doesn't come out of hole. ($T_{water} = 75 \times 10^{-3}$ N/m) $\rho_{water} = 10^3$ kg/m^3, g = 10m/sec^2

Solution :



The lower surface of water, which will try to come out will be spherical. Pressure just outside the spherical surface is :

$P_0 + \rho gh - \frac{2T}{R}\cos\theta = P_0$; $h = \frac{2T}{\rho gR}\cos\theta$

$(h)_{max} = \frac{2T}{\rho gR}(\cos\theta)_{max}$ and $(\cos\theta)_{max} = 1$

So $(h)_{max} = \frac{2T}{\rho gR} = \frac{2\times 75\times 10^{-3}}{10^3\times 10\times 0.05\times 10^{-3}}$ $(h)_{max}$ = 30 cm

SOME OTHER APPLICATIONS OF SURFACE TENSION

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

Solved Example

Example 21. **(Only for JEE Advanced)**

A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Solution : Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the meniscus in the wide and narrow tubes respectively are

$\left(P_1 - \frac{2T}{r_1}\right)$ and $\left(P_2 - \frac{2T}{r_2}\right)$ [excess pressure = $\frac{2T}{r}$].

Difference in these pressures = $\left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference = $P_1 - P_2$

$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$

$= 1.86 \times 10^3 =$ **1860 N/m²**

Example 22. **(Only for JEE Advanced)**

A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0º.

Solution : The surface tension of the liquid is $T = \frac{rh\rho g}{2}$

$= \frac{(0.025\text{cm})(3.0\text{cm})(1.5\text{gm/cm}^3)(980\text{cm/sec}^2)}{2} = 55$ dyne/cm.

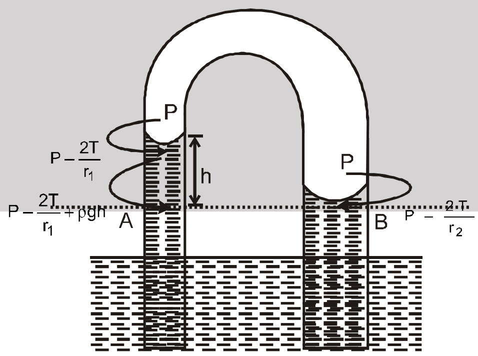
Hence excess pressure inside a spherical bubble

$p = \frac{4T}{R} = \frac{4 \times 55\,\text{dyne/cm}}{(0.5\,\text{cm})}$ = **440 dyne/cm²** .

Example 23. **(Only for JEE Advanced)**

A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm⁻¹. Assume that the angle of contact between water and glass is 0º.

Solution :



Equating pressure at point A and B which are in same horizontal level

$P - \frac{2T}{r_1} + \rho gh = P - \frac{2T}{r_2} \Rightarrow h = \frac{2T}{\rho\ g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Given that $T = 0.07\ Nm^{-1}$, $\rho = 1000\ kgm^{-3}$

$r_1 = \frac{3}{2}$ mm $= \frac{3}{20}$ cm $= \frac{3}{20 \times 100}$ m $= 1.5 \times 10^{-3}$ m, $r_2 = 3 \times 10^{-3}$ m

$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)$ m $= 4.76 \times 10^{-3}$ m = 4.76 mm

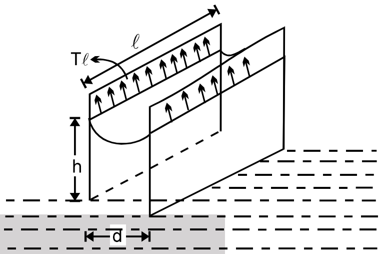
Example 24. **(Only for JEE Advanced)**

Two parallel plates which are separated by a very small distance d, are dipped in water. To how much height will the water raise between the plates (Assume contact angle $\theta \to 0$)

Solution : Lets draw free body diagram of the water raised up . Forces on it are :

(i) The plates pull the surface in upward direction with a force $2T\ell$

(ii) The weight of raised water = $(\rho)(\ell hd)g$ For equilibrium, forces should be balanced.



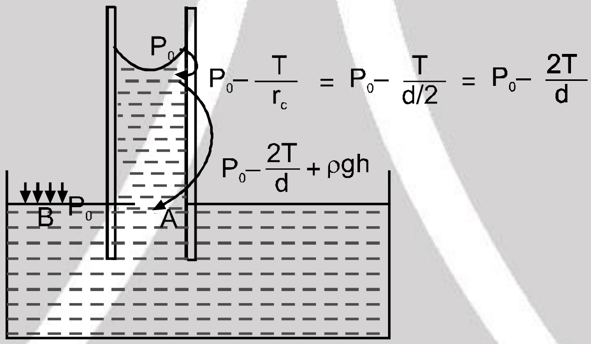
$$2T\ell = (\rho)(\ell hd)g \quad \Rightarrow \quad h = \frac{2T}{\rho gd}$$

Also $= \frac{T}{d/2}$ ρgh ; so we can say that pressure excess due to cylindrical surface = $\frac{T}{d/2} = \frac{T}{r_c}$

pressure excess due to spherical surface= $\frac{2T}{r_c}$

pressure excess due to cylindrical surface = $\frac{T}{r_c}$

Alternative method :



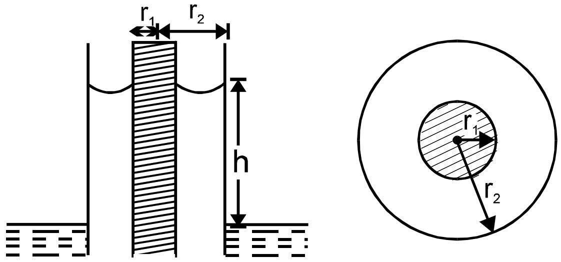
$$\Rightarrow P_0 - \frac{2T}{d} + \rho gh = P_0 \Rightarrow h = \frac{2T}{\rho gd}$$

Example 25. **(Only for JEE Advanced)**

A thin capillary of inner radius r_1 and outer radius r_2 (The inner tube is solid) is dipped in water. To how much height will the water raise in the tube ? (Assume contact angle $\theta \to 0$)

Solution : Applying force balance $T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$

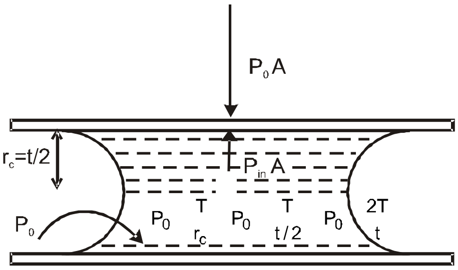


Example 26. **(Only for JEE Advanced)**

A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in Newton.

Solution : Pressure inside the surface

$$P_{in} = P_0 - \frac{T}{r_C} = P_0 - \frac{T}{t/2} = P_0 - \frac{2T}{t},$$



So, net inwards force $= P_0A - P_{in}A = \left(P_0 - \frac{2T}{t}\right)A - P_0A$

$$= \frac{2TA}{t}$$

Here volume between the plates V = A × t

$\Rightarrow \quad t = \frac{V}{A}$ Putting the value of t

$$F = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45 \text{ N};$$

So this much force is required to separate the plates

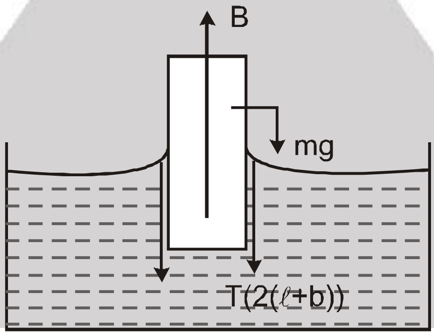
Example 27 **(Only for JEE Advanced)**

A glass plate of length 10 cm, breadth 1.54 cm and thickness 0.20 cm weighs 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water = 73 dyne per cm, g = 980 cm/sec^2.

Solution : The forces acting on the plate are

(i) buoyancive force of water acting upward

$$B = \rho_l V_{sub} g = 1 \times \frac{1.54\times10\times0.2}{2} \times 980 = 1509.2 \text{ dyne.}$$



(ii) Weight of the system acting downward = (8.2) × 980 dyne

(iii) Force of surface tension acting downward = 2 (ℓ + b)T = 2 (10 + 0.2) 73 = 1489.2

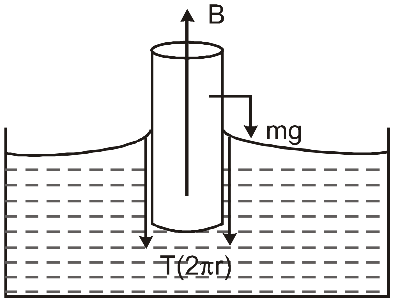
So net downward force = mg + (surface tension force) – B

= (8.2) × 980 + 1489.2 – 1509.2 = 8016.008 dyne = 8.1796 gm force

Example 28. **(Only for JEE Advanced)**

A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Given : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980 cm/sec^2.

Solution : Let ℓ be the length of the tube inside water. The forces acting on the tube are :



(i) buoyancive force of water acting upward

$$B = \pi r^2 \ell \times 1 \times 980 = \frac{22}{7} \times (0.14)^2\ \ell \times 980 = 60.368\ \ell \text{ dyne.}$$

(ii) Weight of the system acting downward = mg = 0.2 × 980 = 196 dyne.

(iii) Force of surface tension acting downward = $2\pi rT$

$$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24 \text{ dyne.}$$

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

$60.368\ \ell = 196 + 64.24 = 260.24.$

$$\therefore \quad \ell = \frac{260.24}{60.368} = \mathbf{4.31\ cm.}$$

Exercise-1

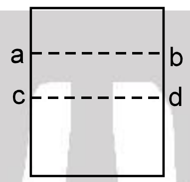
✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Surface tension, Surface energy and capillary rise

A-1. A tube of 1 mm bore is dipped into a vessel containing a liquid of density 0.8 g/cm^3, surface tension 30 dyne/cm and angle of contact zero. Calculate the length which the liquid will occupy in the tube when the tube is held (a) vertical (b) inclined to the vertical at an angle of 30º.

A-2.✍ A soap film is stretched over a rectangular vertical wire frame as shown in the figure, what forces hold section abcd in equilibrium ?



A-3.✍ A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded. (Surface tension of mercury = 32×10^{-2} N/m).

A-4. A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between wires. Surface tension = 72×10^{-3} N/m.

Section (B) : Excess Pressure in drops and bubble

B-1. A soap bubble has radius R and surface tension S, How much energy is required to double the radius without change of temperature.

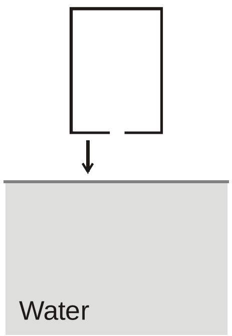
B-2. The work done in blowing a bubble of volume V is W, then what is the work done in blowing a soap bubble of volume 2V ?

B-3. Find the excess pressure inside a drop of mercury of radius 2 mm, a soap bubble of radius 4 mm and an air bubble of radius 4 mm formed inside a tank of water. Surface tension of mercury is 0.465 N/m and soap solution and water are, 0.03 N/m and 0.076 N/m respectively.

B-4_✍ Two identical soap bubbles each of radius r and of the same surface tension T combine to form a new soap bubble of radius R. The two bubbles contain air at the same temperature. If the atmospheric pressure is p_0 then find the surface tension T of the soap solution in terms of p_0, r and R. Assume process is isothermal.

B-5. A spherical drop of water has 1mm radius. If the surface tension of the water is 50×10^{-3} N/m, then find the difference of pressure between inside and outside the spherical drop is :

B-6.✍ An empty container has a circular hole of radius r at its bottom. The container is pushed into water very slowly as shown. To what depth the lower surface of container (from surface of water) can be pushed into water such that water does not flow into the container ?

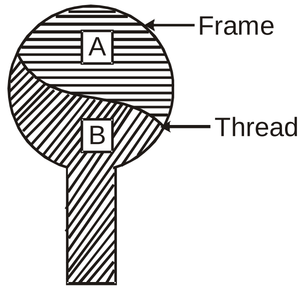


(Surface tension of water = T, density of water = ρ)

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Surface tension, Surface energy and capillary rise

A-1. A thread is tied slightly loose to a wire frame as shown in the figure. And the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread :



(A) becomes convex towards A
(B) becomes concave towards A
(C) remains in the initial position
(D) either (A) or (B) depending on size of A w.r.t. B

A-2. In a surface tension experiment with a capillary tube water rises upto 0.1 m. If the same experiment is repeated in an artificial satellite, which is revolving around the earth ; water will rise in the capillary tube upto a height of :
(A) 0.1 m (B) 0.2 m (C) 0.98 m (D) full length of tube

A-3. A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is :
(A) $2\pi rT + W$ (B) $2\pi rT\cos\theta - W$ (C) $2\pi rT\cos\theta + W$ (D) $W - 2\pi rT\cos\theta$

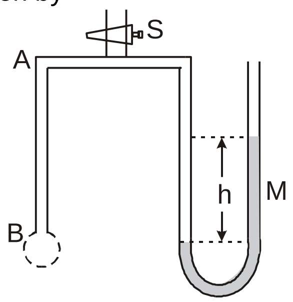
A-4. The surface tension of a liquid is 5 Newton per metre. If a film is held on a ring of area 0.02 metres^2, its surface energy is about :
(A) 5×10^{-2} J (B) 2.5×10^{-2} J (C) 2×10^{-1} J (D) 3×10^{-1} J

A-5. The radii of the two columns is U-tube are r_1 and r_2. When a liquid of density ρ (angle of contact is 0°) is filled in it, the level difference of liquid in two arms is h. The surface tension of liquid is: (g = acceleration due to gravity) :
(A) $\frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$ (B) $\frac{\rho g h (r_2 - r_1)}{2 r_1 r_2}$ (C) $\frac{2(r_2 - r_1)}{\rho g h r_1 r_2}$ (D) $\frac{\rho g h}{2(r_2 - r_1)}$

A-6. Water rises in a capillary tube to a height h. it will rise to a height more than h
(A) on the surface of sun (B) in a lift moving down with an acceleration
(C) at the poles (D) in a lift moving up with an acceleration.

A-7. Insects are able to run on the surface of water because :
(A) insects have less weight
(B) insects swim on water
(C) of the Archimede's upthrust
(D) surface tension makes the surface behave as elastic membrane.

A-8. A tube of fine bore AB is connected to a manometer M as shown. The stop cock S controls the flow of air. AB is dipped into a liquid whose surface tension is σ. On opening the stop cock for a while, a bubble is formed at B and the manometer level is recorded, showing a difference h in the levels in the two arms. if ρ be the density of manometer liquid and r the radius of curvature of the bubble, then the surface tension σ of the liquid is given by



(A) ρhrg (B) $2\rho hgr$ (C) $4\rho hrg$ (D) $\frac{rh\rho g}{4}$

A-9. Two parallel glass plates are dipped partly in the liquid of density 'd' keeping them vertical. If the distance between the plates is 'x', Surface tension for liquid is T & angle of contact is θ then rise of liquid between the plates due to capillary will be :

(A) $\frac{T\cos\theta}{xd}$ (B) $\frac{2T\cos\theta}{xdg}$ (C) $\frac{2T}{xdg\cos\theta}$ (D) $\frac{T\cos\theta}{xdg}$

Section (B) : Excess Pressure in drops and bubble

B-1. When charge is given to a soap bubble, it shows :

(A) a decrease in size (B) no change in size (C) an increase in size

(D) sometimes an increase and sometimes a decreases in size

B-2. A water drop is divided into 8 equal droplets. The pressure difference between the inner and outer side of the big drop will be :

(A) same as for smaller droplet (B) 1/2 of that for smaller droplet

(C) 1/4 of that for smaller droplet (D) twice that for smaller droplet

B-3. An air bubble of radius r in water is at a depth h below the water surface at some instant. If P is atmospheric pressure, d and T are density and surface tension of water respectively, the pressure inside the bubble will be :

(A) $P + h\,dg - \frac{4T}{r}$ (B) $P + h\,dg + \frac{2T}{r}$ (C) $P + h\,dg - \frac{2T}{r}$ (D) $P + h\,dg + \frac{4T}{r}$

B-4. The work done to get n smaller equal size spherical drops from a bigger size spherical drop of water is proportional to :

(A) $\left(\frac{1}{n^{2/3}}\right) - 1$ (B) $\left(\frac{1}{n^{1/3}}\right) - 1$ (C) $n^{1/3} - 1$ (D) $n^{4/3} - 1$

B-5. Two unequal soap bubbles are formed one on each side of a tube closed in the middle by a tap. What happens when the tap is opened to put the two bubbles in communication ?

(A) No air passes in any direction as the pressures are the same on two sides of the tap

(B) Larger bubble shrinks and smaller bubble increases in size till they become equal in size

(C) Smaller bubble gradually collapses and the bigger one increases in size

(D) None of the above

B-6. A soap bubble in vacuum has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal conditions then the radius of the new bubble is :

(A) 2.3 cm (B) 4.5 cm (C) 5 cm (D) 7 cm

B-7. A cylinder with a movable piston contains air under a pressure p_1 and a soap bubble of radius 'r'. The pressure p_2 to which the air should be compressed by slowly pushing the piston into the cylinder for the soap bubble to reduce its size by half will be : (The surface tension is σ, and the temperature T is maintained constant)

(A) $\left[8p_1 + \frac{24\sigma}{r}\right]$ (B) $\left[4p_1 + \frac{24\sigma}{r}\right]$ (C) $\left[2p_1 + \frac{24\sigma}{r}\right]$ (D) $\left[2p_1 + \frac{12\sigma}{r}\right]$

B-8. A vessel whose bottom has round holes with a diameter of d = 0.1 mm is filled with water. The maximum height of the water level h at which the water does not flow out, will be : (The water does not wet the bottom of the vessel). [S.T of water = 70 dyn/cm]

(A) h = 24.0 cm (B) h = 25.0 cm (C) h = 26.0 cm (D) h = 28.0 cm

PART - III : MATCH THE COLUMN

1.

Column - I	**Column - II**
(A) Spliting of brigger drop into small drops	(P) Temperature changes
(B) Formation of bigger drop from small drops.	(Q) Temprerature remain constant
(C) Spraying of liquid	(R) Surface energy changes
(D) Spliting of bigger soap bubble into small soap bubble of same thickness	(S) Surface energy remain unchange

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

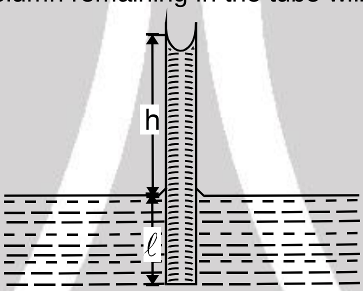
1. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is punctured inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the soap solution be T, then the tension in the thread will be :

(A) $\pi R^2/T$ (B) $\pi R^2 T$ (C) $2\pi RT$ (D) $2RT$

2. A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in capillary tube will be

(A) 2M (B) M (C) $\frac{M}{2}$ (D) 4M

3. Water rises to a height h in a capillary tube lowered vertically into water to a depth ℓ as shown in the figure. The lower end of the tube is now closed, the tube is then taken out of the water and opened again. The length of the water column remaining in the tube will be :



(A) 2h if $\ell > h$ and $\ell + h$ if $\ell < h$ (B) h if $\ell > h$ and $\ell + h$ if $\ell < h$

(C) 4h if $\ell > h$ and $\ell - h$ if $\ell < h$ (D) h/2 if $\ell > h$ and $\ell + h$ if $\ell < h$

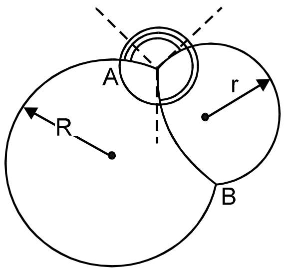
4. A soap bubble of radius r_1 is placed on another soap bubble of radius $r_2 (r_1 < r_2)$. The radius R of the soapy film separating the two bubbles is :

(A) $r_1 + r_2$ (B) $\sqrt{r_1^2 + r_2^2}$ (C) $(r_1^3 + r_2^3)$ (D) $\frac{r_2 r_1}{r_2 - r_1}$

5. The high domes of ancient buildings have structural value (besides beauty). It arises from pressure difference on the two faces due to curvature (as in soap bubbles). There is a dome of radius 5 m and uniform (but small) thickness. The 'surface tension' of its masonary structure is about 500 N/m. Treated as hemispherical, the maximum load the dome can support is nearest to

(A) 1500 kg wt. (B) 3000 kg wt. (C) 6000 kg wt. (D) 12000 kg wt.

6. A soap - bubble with a radius 'r' is placed on another bubble with a radius R (figure). Angles between the films at the points of contact will be –



(A) 120° (B) 30° (C) 45° (D) 90°

7. A large number of liquid drops each of radius 'a' coalesce to form a single spherical drop of radius 'b'. The energy released in the process is converted into kinetic energy of the big drop formed. The speed of big drop will be :

(A) $\sqrt{\frac{6T}{\rho}\left[\frac{1}{a}-\frac{1}{b}\right]}$ (B) $\sqrt{\frac{4T}{\rho}\left[\frac{1}{a}-\frac{1}{b}\right]}$ (C) $\sqrt{\frac{8T}{\rho}\left[\frac{1}{a}-\frac{1}{b}\right]}$ (D) $\sqrt{\frac{5T}{\rho}\left[\frac{1}{a}-\frac{1}{b}\right]}$

8. At critical temperature, the surface tension of a liquid :

(A) is zero (B) is infinity

(C) is same as that at any other temperature (D) cannot be determined

9._ The excess pressure inside a soap bubble is equal to 2 mm of kerosene (density 0.8 g cm^{-3}). If the diameter of the bubble is 3.0 cm, the surface tension of soap solution is **[Olympiad (Stage-1) 2017]**

(A) 39.2 dyne cm^{-1} (B) 45.0 dyne cm^{-1} (C) 51.1 dyne cm^{-1} (D) 58.8 dyne cm^{-1}

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. There is a soap bubble of radius 2.4×10^{-4} m in air cylinder which is originally at the pressure 10^5 N/m^2. The air in the cylinder is now compressed isothermally until the radius of the bubble is halved. Calculate now the pressure (in atm) of air in the cylinder. The surface tension of the soap solution is 0.08 N/m

2. A long capillary tube of radius r = 1 mm open at both ends is filled with water and placed vertically. What will be the height (in cm) of the column of water left in the capillary in nearest integer ? The thickness of the capillary walls is negligible. (Surface tension of water is 72 dyne/cm and g = 1000 cm/sec^2)

3. Two spherical soap bubbles collapses. If V is the consequent change in volume of the contained air and S is the change in the total surface area and T is the surface tension of the soap solution, then if relation between P_0, V, S and T are $\lambda P_0V + 4ST = 0$, then find λ ? (if P_0 is atmospheric pressure) : Assume temperature of the air remain same in all the bubbles

4. A capillary of 1mm diameter, is dipped vertically in a pot of water. If gauge pressure of the water in the tube 5.0 cm below the surface is 2λ N/m^2 then find λ. Surface tension of water = 0.075 N/m. (take g = 9.8 m/s^2 and ρ_w = 1000 kg/m^3)

5. A capillary tube with very thin walls is attached to the beam of a balance which is then equalized. The lower end of the capillary is brought in contact with the surface of water after which an additional load of P = 0.135 gm force is needed to regain equilibrium. If the radius of the capillary is $\frac{\lambda}{10}$ mm then find λ. The surface tension of water is 70 dyn/cm. (g = 9.8 m/s^2)

6. A capillary tube sealed at the top has an internal radius of r = 0.05 cm. The tube is placed vertically in water, with its open end dipped in water. Find greatest integer corresponding to the length (in meter) of such a tube be for the water in it to rise in these conditions to a height h = 1 cm ? The pressure of the air is P_0 = 1 atm. = 76 cm of Hg, density of Hg = 13.6 g/cm^3, g = 9.8 m/sec^2. The surface tension of water is σ = 70 dyn/cm. (Assume temperature of air in the tube is constant)

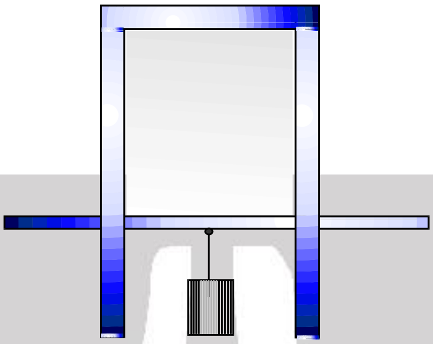
7. A cube with mass m = 20 g wettable by water floats on the surface of water. Each face of the cube is 3 cm long. If the distance (cm) between the lower face of the cube and the surface of the water in contact with the cube is 4.6/ λ cm, then find λ ? [S.T of water α = 70 dyn/cm, assume contact angle to be $\theta = 0^o$]

8. The end of a capillary tube with a radius r is immersed into water. What amount of heat will be evolved when the water rises in the tube ? If surface tension of water 'T' density of water = ρ. Given $\frac{T^2}{\rho g} = \frac{2}{\pi}$

9. A soap bubble of radius 'r' and surface tension 'T' is given a potential of 'V' volt. If the new radius 'R' of the bubble is related to its initial radius by equation, $P_0\,[R^3 - r^3] + \lambda T\,[R^2 - r^2] - \varepsilon_0 V^2 R/2 = 0$, where P_0 is the atmospheric pressure. Then find λ

10. A glass rod of diameter d_1 = 1.5 mm is inserted symmetrically into a glass capillary with inside diameter d_2 = 2.0 mm. Then the whole arrangement is vertically oriented and brought in contact with the surface water To what height (cm) will the liquid rise in the capillary. Surface tension of water = 73×10^{-3} N/m, Angle of contact = 0º. (Use g = 9.8 m/s²)

11. A rectangular wire frame with one movable side is covered by a soap film (fig.). What work (erg) will be done if this side of the frame is moved a distance S = 2 mm ? The length of the movable side is ℓ = 6cm. The surface tension of the soap film is α = 40 dyn/cm.



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. When a capillary tube is dipped in a liquid, the liquid rises to a height h in the tube. The free liquid surface inside the tube is hemispherical in shape. The tube is now pushed down so that the height of the tube outside the liquid is less than h :
(A) the liquid will ooze out of the tube slowly
(B) the liquid will come out of the tube like in a small fountain
(C) the free liquid surface inside the tube will not be hemispherical
(D) the liquid will fill the tube but not come out of its upper end

2. When a capillary tube is immersed into a liquid, the liquid neither rises nor falls in the capillary ?
(A) The angle of contact must be 90° (B) The angle of contact may be 90°
(C) The surface tension of liquid must be zero (D) The surface tension of liquid may be zero

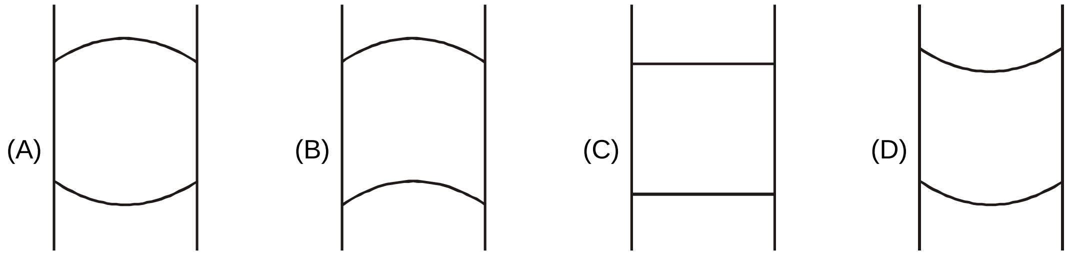
3. Angle of contact between a liquid and a solid is a property of :
(A) the material of liquid (B) the material of solid
(C) the mass of the solid (D) the shape of the solid

4. When a drop splits up into number of drops:
(A) total surface area increases (B) volume increases
(C) energy is absorbed (D) energy is liberated

5. If a liquid rises to same height in two capillaries of same material at same tempreture then.
(A) Weight of liquid in both capillaries will be equal
(B) Radius of miniscus will be equal
(C) For this capillaries must be curve and vertical.
(D) Hydrostatic pressure at the bare of capillaries must be same.

6. When a glass capillary tube is dipped in a liquid, then liquid rises to a height h in the tube. The free liquid surface inside the tube is hemispherical. The tube is now pushed down so that the height of the tube outside the liquid is less than h. Then
(A) The liquid will come out of the tube
(B) The liquid will fill the tube but not come out of its upper end
(C) The free liquid surface inside tube may be concave
(D) The free liquid surface inside tube may be convex.

7. A vertical glass capillary tube, open at both ends, contains some water. Which of following shapes may not be possible ?



8. The rise of liquid in a capillary tube depends on.
(A) The material (B) The length (C) Outer radius (D) Inner radius

9. Suppose outside pressure is P_0 and surface tension of soapwater solution is T and we are blowing a soap bubble of radius R. Then
(A) Pressure inside soap babble of radius R will be $P_0 + \frac{4T}{R}$.
(B) Pressure inside soap bubble of radius R will be $P_0 + \frac{2T}{R}$
(C) work done by external agent to blow soap bubble is equal to summation of work done against increase pressure from P_0 to $(P_0 + \frac{4T}{R})$ and work done against increase in surface energy.
(D) None of these

10. If for a liquid in a vessel, force of a cohesion is twice of adhesion:
(A) the meniscus will be convex upwards (B) the angle of contact will be obtuse
(C) the liquid will descend in the capillary tube (D) the liquid will wet the solid

PART - IV : COMPREHENSION

Comprehension - 1

The internal radius of one limb of a capillary U-tube is r_1 = 1 mm and the internal radius of the second limb is r_2 = 2 mm. The tube is filled with some mercury, and one of the limbs is connected to a vacuum pump. The surface tension & density of mercury are 480 dyn/cm & 13.6 gm/cm^3 respectively. (Assume contact angle to be $\theta = 180^o$) (g = 9.8 m/s^2)

1. What will be the difference in air pressure when the mercury levels in both limbs are at the same height ?
(A) 3.53 mm of Hg (B) 1.51 mm of Hg (C) 0.51 mm of Hg (D) 5.52 mm of Hg

2. Which limb of the tube should be connected to the pump ?
(A) Limb having radius 2 mm (B) Limb having radius 1mm
(C) Any of the limb (D) None of these

Comprehension - 2

An open capillary tube contains a drop of water. The internal diameter of the capillary tube is 1mm. Determine the radii of curvature of the upper and lower meniscuses in each case. Consider the wetting to be complete. Surface tension of water = 0.073 N/m. (g = 9.8 m/s^2)

3. When the tube is in its vertical position, the drop forms a column with a length of 2 cm.
(A) 0.5 mm, 1.52 mm (B) 0.5 mm, 1.46 mm
(C) 0.5 mm, lower surface will be flat (D) 0.4 mm, 1.46 mm

4. When the tube is in its vertical position, the drop forms a column with a length of 4 cm.
(A) 0.5 mm, 1.52 mm (B) 0.5 mm, 1.46 mm
(C) 0.5 mm, lower surface will be flat (D) 0.4 mm, 1.46 mm

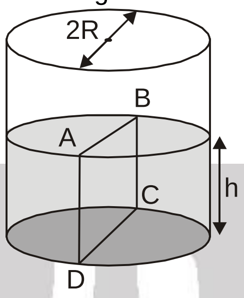
5. When the tube is in its vertical position, the drop forms a column with a length of 2.98 cm.
(A) 0.5 mm, 1.52 mm (B) 0.5 mm, 1.46 mm
(C) 0.5 mm, lower surface will be flat (D) 0.4 mm, 1.46 mm

Exercise-3

Marked Questions can be used as Revision Questions.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

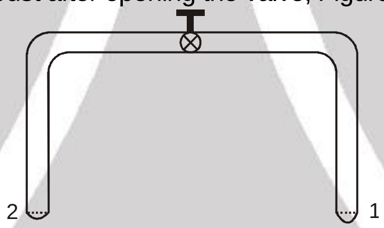
1. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **[JEE 2007, 3/184]**



(A) $\left|2P_0Rh + \pi R^2\rho gh - 2RT\right|$ (B) $\left|2P_0Rh + R\rho gh^2 - 2RT\right|$

(C) $\left|P_0\pi R^2 + R\rho gh^2 - 2RT\right|$ (D) $\left|P_0\pi R^2 + R\rho gh^2 + 2RT\right|$

2. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, Figure : **[JEE -2008 3/163, –1]**



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(C) no change occurs
(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases.

3. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2cm and 4cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.] **[IIT 2009_4/160, –1]**

Paragraph for questions 4 to 6

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

4. If the radius of the opening of the dropper is r; the vertical force due to the surface tension on the drop of radius R (assuming r << R) is : **[IIT 2010; 3/163, –1]**

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2T}{R}$ (D) $\frac{2\pi R^2T}{r}$

5. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, g = 10 ms^{-2}, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately : **[IIT 2010; 3/163, –1]**

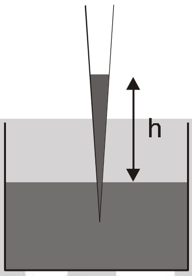
(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

6. After the drop detaches, its surface energy is : **[IIT 2010; 3/163, –1]**

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

7. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'.The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$ where 'k' is a constant. Then N is **[JEE-2011, 4/160]**

8. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)
[JEE (Advanced)-2014, 3/60, –1]



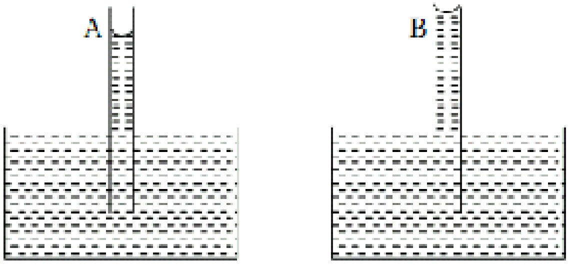
(A) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$ (B) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$ (C) $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$ (D) $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$

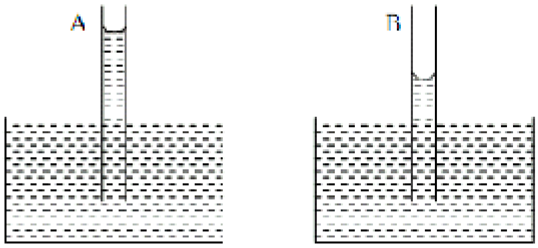
9. A drop of liquid of radius R = 10^{-2} m having surface tension $S = \frac{0.1}{4\pi}Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is : **[JEE (Advanced) 2017, 3/61]**

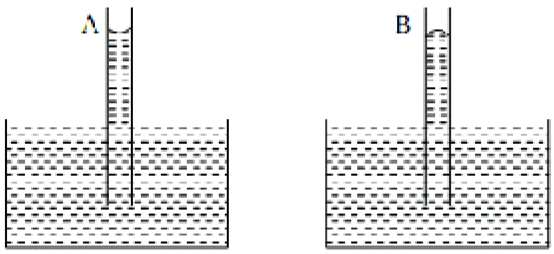
10*. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true? **[JEE (Advanced) 2018, P-1, 4/60, –2]**
(A) For a given material of the capillary tube, h decreases with increase in r
(B) For a given material of the capillary tube, h is independent of σ
(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases
(D) h is proportional to contact angle θ

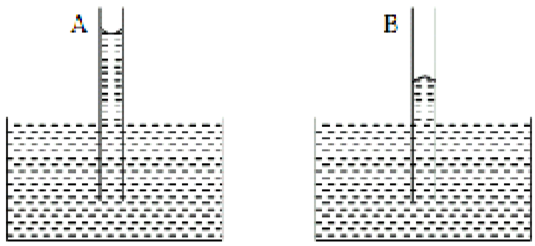
PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A capillary tube (1) is dipped in water. Another identical tube (2) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes ? **[AIEEE 2008, 4/120, –1]**

(1) 

(2) 

(3) 

(4) 

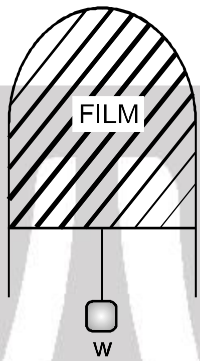
2. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly. (Surface tension of soap solution = $0.03\ Nm^{-1}$) **[AIEEE - 2011, 4/120, –1]**

(1) 4π mJ (2) 0.2π mJ (3) 2π mJ (4) 0.4π mJ

3. Two mercury drops (each of radius 'r') merge to from bigger drop. The surface energy of the bigger drop, if T is the surface tension, is : **[AIEEE 2011, 4/120, –1]**

(1) $4\pi r^2 T$ (2) $2\pi r^2 T$ (3) $2^{8/3}\pi r^2 T$ (4) $2^{5/3}\pi r^2 T$

4. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is : **[AIEEE 2012, 4/120, –1]**

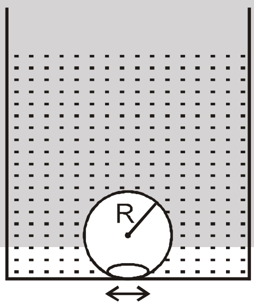


(1) $0.0125\ Nm^{-1}$ (2) $0.1\ Nm^{-1}$ (3) $0.05\ Nm^{-1}$ (4) $0.025\ Nm^{-1}$

5. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible ? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[JEE (Main) 2013, 4/120,–1]**

(1) $\rho L/T$ (2) $\sqrt{T/\rho L}$ (3) $T/\rho L$ (4) $2T/\rho L$

6. On heating water, bubbles beings formed at the bottom of the vessel detatch and rise. Take the bubles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of water is T, value of r just before bubles detatch is : (density of water is ρ_w) **[JEE (Main) 2014,4/120,–1]**



(1) $R^2\sqrt{\dfrac{\rho_w g}{3T}}$ (2) $R^2\sqrt{\dfrac{\rho_w g}{6T}}$ (3) $R^2\sqrt{\dfrac{\rho_w g}{T}}$ (4) $R^2\sqrt{\dfrac{3\rho_w g}{T}}$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. (a) 1.53 cm, (b) 1.77 cm

A-2. Surface tension forces F_{ab}, F_{cd} and weight. Equilibrium only when $F_{ab} > F_{cd}$ and this is due to difference in concentration of soap solution in film.

A-3. 3.98×10^{-2} J **A-4.** 1.44×10^{-5} J

Section (B) :

B-1. $24\pi R^2 S$ **B-2.** $2^{2/3}$ W

B-3. (a) 465 N/m^2 (b) 30 N/m^2 (c) 38 N/m^2

B-4 $T = \dfrac{p_0(2r^3 - R^3)}{4(R^2 - 2r^2)}$ **B-5.** 100N/m^2

B-6. $\dfrac{2T}{\rho g r}$

PART - II

Section (A) :

A-1.	(B)	**A-2.**	(D)	**A-3.**	(C)
A-4.	(C)	**A-5.**	(A)	**A-6.**	(B)
A-7.	(D)	**A-8.**	(D)	**A-9.**	(B)

Section (B) :

B-1.	(C)	**B-2.**	(B)	**B-3.**	(B)
B-4.	(C)	**B-5.**	(C)	**B-6.**	(C)
B-7.	(A)	**B-8.**	(D)		

PART - III

1. (A) – P,R ; (B) – P,R ; (C) – P,R ;(D) –Q,S

EXERCISE-2

PART - I

1.	(D)	**2.**	(A)	**3.**	(A)
4.	(D)	**5.**	(B)	**6.**	(A)
7.	(A)	**8.**	(A)	**9.**	(D)

PART - II

1.	8	**2.**	3	**3.**	3
4.	98	**5.**	15	**6.**	5
7.	2	**8.**	4	**9.**	4
10.	6	**11.**	96		

PART - III

1.	(CD)	**2.**	(BD)	**3.**	(AB)
4.	(AC)	**5.**	(AB)	**6.**	(BCD)
7.	(ABC)	**8.**	(ABD)	**9.**	(AC)
10.	(ABC)				

PART - IV

1.	(A)	**2.**	(B)	**3.**	(A)
4.	(B)	**5.**	(C)		

EXERCISE-3

PART - I

1.	(B)	**2.**	(B)	**3.**	6
4.	(C)	**5.**	(A)	**6.**	(B)
7.	**3**	**8.**	(D)	**9.**	6
10.	(AC)				

PART - II

1.	(2)	**2.**	(4)	**3.**	(3)
4.	(4)	**5.**	(4)	**6.**	(Bonus)

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A capillary of radius r is lowered into a wetting agent with surface tension α and density d. Determine the height h_0 to which the liquid will rise in the capillary. Calculate the work done by surface tension and the potential energy acquired by the liquid in the capillary and compare the two. Explain the difference in the results obtained.

2. A U-tube is made up of capillaries of bores 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne/cm and zero contact angle. Calculate the density of the liquid if the difference in the levels of the meniscus is 1.25 cm.

3. A film of soap solution is formed on a loop frame loop of 6.28 cm long thread is gently put on the film and the film is broken with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution = 0.030 N/m.

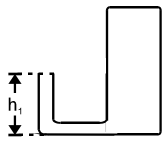
4. What is the excess pressure inside a bubble of soap solution of radius 5.0 mm, given that the surface tension of soap solution is 2.5×10^{-2} N/m. If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.2) what would be pressure inside the bubble. [1 atm = 1.01×10^5 N/m^2]

5. A mercury drop shaped as round tablet of radius 'R' and thickness 'h' is located between two horizontal glass-plates. Assuming h << R, find the expression in weight which has placed on the upper plate to diminish the distance between the plates 'n' times. The angle of contact = θ. Calculate the weight if R = 2.0 cm, h = 0.38 mm, n = 2 and θ = 135º. Surface tension of Hg = 0.49 N/m.

6. The lower end of a capillary of radius r = 0.2 mm and length ℓ = 8 cm is immersed in water whose temperature is constant and equal to T_{low} = 0ºC. The temperature of the upper end of the capillary is T_{up} = 100ºC. Determine the height h to which the water in the capillary rises, assuming that the thermal conductivity of the capillary is much higher than the thermal conductivity of water in it. The heat exchange with the ambient should be neglected.
Use the following temperature dependence of the surface tension of water :

T, ºC	0	20	50	90
σ, mN/m	76	73	67	60

7. A capillary tube of radius r and height h_1 is connected to a broad tube as shown in fig. The broad tube is gradually filled with drops of water falling at equal intervals. Plot the changes in the levels of the water in both tubes with time and changes in the difference between these levels. Calculate the maximum water level in the broad-tube and maximum difference in levels. The surface tension of water is α.



8. If the radius and surface tension of a spherical soap bubble be 'R' & 'T' respectively, show that the charge required to double its radius would be, $8\pi R\ [\varepsilon_0 R[7\ PR + 12\ T]]^{1/2}$. (Where P is the atmospheric pressure and process isothermal)

9. A conical glass capillary tube A of length 0.1 m has diameters 10^{-3} m and 5×10^{-4} m at the ends. When it is just immersed in a liquid at 0ºC with larger diameter in contact with it, the liquid rises to 8×10^{-2} m in the tube. In another cylindrical glass capillary tube B, when immersed in the same liquid at 0ºC, the liquid rises to 6×10^{-2} m height. The rise of liquid in the tube B is only 5.5×10^{-2} m when the liquid is at 50ºC. Find the rate at which the surface tension changes with temperature, considering the change to be linear. The density of liquids is $(1/14) \times 10^4$ kg/m^3 and the angle of contact is zero. (Effect of temperature on the density of liquid and glass is negligible). (g = 9.8 N/kg.) **[REE - 1994]**

10. The limbs of a manometer consists of uniform capillary tubes of radii 1.44×10^{-3} m and 7.2×10^{-4} m. Find out the correct pressure difference if the level of the liquid in the narrower tube stands 0.2 m above that in the broader tube. (density = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m). (take g = 9.8 m/s^2) **[REE - 1985]**

11. A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. Tube is immersed vertically into a liquid of surface tension 5.06×10^{-2} N/m. To what length has the capillary to be immersed so that the liquid level inside and outside the capillary becomes the same. What will happen to the water level inside the capillary if the seal is now broken. Assume isothermal condition in the tube. (Use g = 10 m/s^2) **[REE - 1993]**

12. Find the attraction force between two parallel glass plates separated by a distance h = 0.10 mm, after a water drop of mass m = 70 mg was introduced between them. The wetting is assumed to be complete.

13. Two vertical plates submerged, partially in a wetting liquid form a wedge with a very small angle $\delta\varphi$. The edge of this wedge is vertical. The density of the liquid is ρ, its surface tension is α, the contact angle is θ. Find the height h, to which the liquid rises, as a function of the distance x from the edge.

14. A water drop falls in air with a uniform velocity. Find the difference between the curvature radii of the drops surface at the upper and lower points of the drop separated by the distance h = 2.3 mm.

15. Bubbles are made by dipping a circular ring of radius b in a soap solution and then blowing air on the film formed on the ring. Assume that the blown air is in the form of a cylinder of radius b. It has speed v and stops after striking the surface of the bubble being formed. The bubble grows spherically. Let the radius R of the bubble (>> b), so that the air strikes the bubble surface perpendicularly. The surface tension of the solution is T and air density is ρ. Obtain the radius of the bubble when it separates from the ring in terms of the given parameters (neglect the mass of the bubble). **[JEE 2003, 4/60]**

HLP Answers

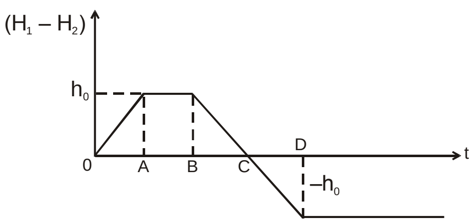
1. $\left[\frac{2\alpha}{dgr}, \frac{4\pi\alpha^2}{dg}, \frac{2\pi\alpha^2}{dg}\right]$ **2.** 0.7991 g/cm^3

3. 6×10^{-4} N

4. 20 N/m^2, 1.05714×10^5 N/m^2

5. 0.7 kg **6.** 6.4 cm

7.



Maximum height = $h_1 + h_0$

Maximum difference in level = h_0 where $h_0 = \frac{2\alpha}{dgr}$.

9. -1.4×10^{-4} N/(m – ºC)

10. 1860 N/m^2

11. ≈ 0.01 m, liquid will rise to top and radius of meniscus will be 1.012×10^{-4} m

12. $F \approx 2\alpha m/\rho h^2 = 1.0$ N

13. $h = 2a\cos\theta / \rho g x \delta\varphi$

14. $R_2 - R_1 \approx 1/8 \rho g h^3/\alpha = 0.20$ mm

15. $\frac{4T}{\rho v^2}$

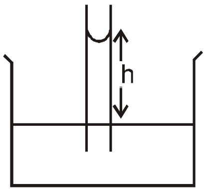
HINT & SOLUTION OF SURFACE TENSION

EXERCISE-1

PART - I

भाग - I

A-1.



(a) for vertically placed tube :

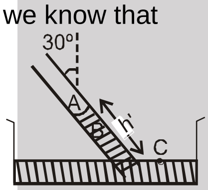
$$h = \frac{2T\cos\theta}{r\rho g}$$

where r = 0.5 mm.

so $$h = \frac{2\times 30\,\frac{10^{-5}}{10^{-2}}\left(\frac{N}{m}\right)\times\cos\theta}{0.5\times10^{-3}\ \ m\ \ \times 800\ \frac{Kg}{m^3}\times 9.8\left(\frac{m}{sec^2}\right)} = 1.51 \text{ cm.}$$

(b) for inclined tube :

we know that



$P_A = P_0$

$P_C = P_0$

and $P_B = P_0 - \frac{2T}{R}$ where P_0 = atmospheric pressure

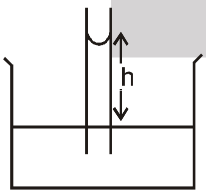
Equating pressure of the level in liquid : we get

$$P_0 - \frac{2T}{R} + (h'\cos 30^o)\rho g = P_0.$$

$$\Rightarrow h' = \frac{2T}{(\cos 30^o)\ R\rho g} = \frac{2T\ \cos\theta}{(\cos 30^o)\ r\rho g} = \frac{h}{\cos 30^o}$$

where h was found in part (a) so $h' = \frac{h}{\cos 30^o}$ = 1.749 cm.

Sol.



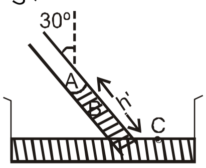
(a) उर्ध्वाधर रखी हुई नली के लिये

$$h = \frac{2T\cos\theta}{r\rho g}$$

जहाँ r = 0.5 mm.

इसलिए $$h = \frac{2\times 30\,\frac{10^{-5}}{10^{-2}}\left(\frac{N}{m}\right)\times\cos\theta}{0.5\times10^{-3}\ \ m\ \ \times 800\ \frac{Kg}{m^3}\times 9.8\left(\frac{m}{sec^2}\right)} = 1.51 \text{ cm.}$$

(b) झुकी हुई नली के लिये :



हम जानते है कि

$P_A = P_0$

$P_C = P_0$

तथा $P_B = P_0 - \dfrac{2T}{R}$

जहाँ P_0 = वायुमण्डलीय दाब

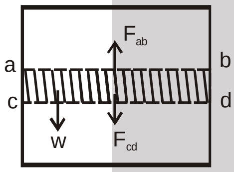
द्रव में समतल का दाब बराबर रखने पर हम पाते हैं

$$P_0 - \frac{2T}{R} + (h' \cos 30^o)\rho g = P_0.$$

$$\Rightarrow h' = \frac{2T}{(\cos 30^o)\ R\rho g} = \frac{2T\ \cos\theta}{(\cos 30^o)\ r\rho g} = \frac{h}{\cos 30^o}$$

जहाँ h भाग (a) में प्राप्त किया था, इसलिए $h' = \dfrac{h}{\cos 30^o} = 1.749$ cm.

A-2.



The FBD of part abcd is as shown. The surface tension forces F_{ab} upward; F_{cd} downward and weight (w) of part abcd are holding the part abcd in equilibrium.

$F_{ab} - F_{cd} = w$.

Clearly $F_{ab} > F_{cd}$ and this is due to difference in concentration of soap solution in film.

चित्र में दिखाये अनुसार भाग abcd का मुक्त वस्तु चित्र (FBD) है पृष्ठतनाव बल F_{ab} ऊपर की तरफ, F_{cd} नीचे की तरफ तथा भाग a,b,c,d का भार ये सब इस भाग को साम्यावस्था में उठाये हुए हैं।

$F_{ab} - F_{cd} = w$.

स्पष्टतः $F_{ab} > F_{cd}$ तथा यह फिल्म में साबुन के घोल में सान्द्रता अन्तर के कारण है।

A-3. We know that $dw = T\,dA \quad \Rightarrow \quad \Delta W = T\,\Delta A$

$\therefore$ in drops, only one surface area is formed.

and $\Delta A = 10^6 \times 4\pi r^2 - 4\pi R^2. \quad = 4\pi R^2\,[100 - 1]$

so $\Delta W = \left(32 \times 10^{-2}\dfrac{N}{m}\right) \times 4\pi\,(10^{-2}\text{ m})^2\,[99] \quad = 3.978 \times 10^{-2}$ J

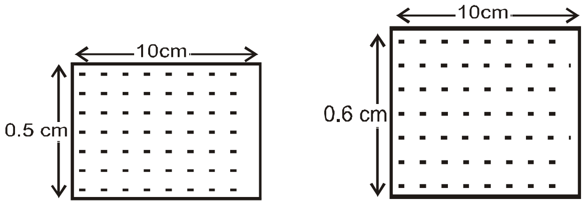
हम जानते हैं कि $dw = T\,dA \quad \Rightarrow \quad \Delta W = T\,\Delta A$

$\therefore$ बून्द में केवल एक ही पृष्ठ क्षेत्र बनेगा

और $\Delta A = 10^6 \times 4\pi r^2 - 4\pi R^2. \quad = 4\pi R^2\,[100 - 1]$

इसलिये $\Delta W = \left(32 \times 10^{-2}\dfrac{N}{m}\right) \times 4\pi\,(10^{-2}\text{ m})^2\,[99] = 3.978 \times 10^{-2}$ J

A-4.



The process is shown in the figure. As we have to produce 2 films; so

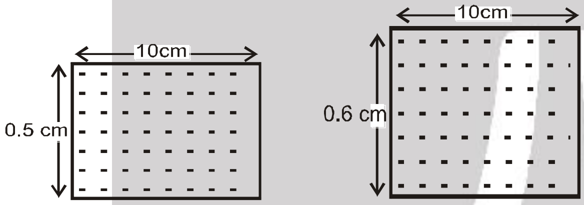
$\Delta W = (2\ \Delta A)\ T$

$= 2\ [10\text{ cm} \times 0.6\text{ cm} - 10\text{ cm} \times 0.5\text{ cm}] \times 72 \times 10^{-3}\ \dfrac{N}{m}$

$= 2 \times (10 \times 10^{-2}\text{ m})\ (0.1 \times 10^{-2}\text{ m}) \times 72 \times 10^{-3}\ \dfrac{N}{m}$

$= 1.44 \times 10^{-5}$ J

Sol.



प्रक्रिया चित्र में दर्शायी गयी है। जैसे कि हम को दो फिल्म बनानी है, इसलिये

$\Delta W = (2\ \Delta A)\ T$

$= 2\ [10\text{ cm} \times 0.6\text{ cm} - 10\text{ cm} \times 0.5\text{ cm}] \times 72 \times 10^{-3}\ \dfrac{N}{m}$

$= 2 \times (10 \times 10^{-2}\text{ m})\ (0.1 \times 10^{-2}\text{ m}) \times 72 \times 10^{-3}\ \dfrac{N}{m}$

$= 1.44 \times 10^{-5}$ J

B-1. On doubling the radius,

$\Delta A = 4\pi\ (2\ R)^2 - 4\ \pi R^2 = 12\pi R^2$

but $\Delta W = (2 \times \Delta A) \times S = 24\ \pi R^2\ S.$

त्रिज्या दो गुनी करने पर ,

$\Delta A = 4\pi\ (2\ R)^2 - 4\ \pi R^2 = 12\pi R^2$

लेकिन $\Delta W = (2 \times \Delta A) \times S = 24\ \pi R^2\ S.$

B-2. Given $W = 2 \times (4\ \pi\ R^2) \times T$

where $\dfrac{4}{3}\pi R^3 = V$ ....(i)

and we want of find $W' = 2 \times (4\pi)\ (R')^2 \times T$

where $\dfrac{4}{3}\pi\ (R')^3 = 2V$ . ...(ii)

Dividing equs. (i) and (ii)

$\dfrac{R}{R'} = \dfrac{1}{2^{\frac{1}{3}}} \quad \Rightarrow \quad R' = 2^{\frac{1}{3}}\ R.$

So $W' = 2T \times 4\ \pi \times 2^{\frac{2}{3}} \cdot R^2 = 2^{\frac{2}{3}}\ (4\ \pi\ 2\ T\ R^2) = 2^{\frac{2}{3}}\ W.$

दिया गया है $W = 2 \times (4\ \pi\ R^2) \times T$

जहाँ $\dfrac{4}{3}\pi R^3 = V.$ ....(i)

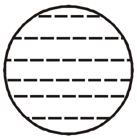
तथा हम प्राप्त करना चाहते हैं $W' = 2 \times (4\pi)\ (R')^2 \times T$

जहाँ $\dfrac{4}{3}\ \pi\ (R')^3 = 2V$ . ...(ii)

समीकरण (i) तथा (ii) को विभाजित करने पर

$$\frac{R}{R'} = \frac{1}{2^{\frac{1}{3}}} \quad \Rightarrow \quad R' = 2^{\frac{1}{3}} R.$$

इसलिये $W' = 2T \times 4\pi \times 2^{2/3} \cdot R^2 = 2^{2/3}(4\pi\, 2\, T\, R^2) = 2^{2/3} W.$

B-3. (a)  drop of r = 2 mm.

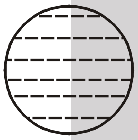
$$P_{excess} = \frac{2T}{R} = \frac{2\times 0.465}{2\times 10^{-3}\ m}\frac{N}{m} = 465\ \frac{N}{m^2}$$

(b) Soap bubble has 2 films :

$$\text{so } P_{excess} = \frac{4T}{R} = \frac{4\times 0.03\frac{N}{m}}{4\times 10^{-3}\ m} = 30\ \frac{N}{m^2}$$

(c) As the air bubble is being formed inside a tank of water; so only one layer is formed.

$$P_{excess} = \frac{2T}{R} = \frac{2\times 0.076\frac{N}{m}}{4\times 10^{-3}\ m} = 38\ \frac{N}{m^2}$$



(a) r = 2 mm की बून्द

$$P_{\text{आधिक्य}} = \frac{2T}{R} = \frac{2\times 0.465}{2\times 10^{-3}\ m} = 465\ \frac{N}{m^2}$$

(b) साबुन के बुलबुले में दो सतह होती है :

$$\text{इसलिये } P_{\text{अतिरिक्त}} = \frac{4T}{R} = \frac{4\times 0.03\frac{N}{m}}{4\times 10^{-3}\ m} = 30\ \frac{N}{m^2}$$

(c) क्योंकि वायु का बुलबुला टंकी के पानी के अन्दर बना है इसलिए इसकी केवल एक ही सतह बनेगी।

$$P_{\text{अतिरिक्त}} = \frac{2T}{R} = \frac{2\times 0.076\frac{N}{m}}{4\times 10^{-3}\ m} = 38\ \frac{N}{m^2}$$

B-4 Total number of moles of air in the two soap bubbles = number of moles of air in the resulting bubble.
दो साबुन के बुलबुलों में हवा के मोलों की संख्या = परिणामी बुलबुले में हवा के मोलों की संख्या

$$\frac{2pv}{RT} = \frac{p'v'}{RT} \qquad 2pv = p'v'$$

$$2\left(p_0 + \frac{4T}{r}\right)\frac{4}{3}\pi r^3 = \left(p_0 + \frac{4T}{R}\right)\frac{4}{3}\pi R^3$$

$$2\left(p_0 + \frac{4T}{r}\right)r^3 = \left(p_0 + \frac{4T}{R}\right)R^3 \qquad \therefore \qquad T = \frac{p_0(R^3 - 2r^3)}{8r^2 - 4R^2} = \frac{p_0(R^3 - 2r^3)}{4(2r^2 - R^2)}$$

B-5. $$P_{excess\ \text{आधिक्य}} = \frac{2T}{R} = \frac{2\left(50\times 10^{-3}\right)}{(10^{-3})} = 100\ N/m^2$$

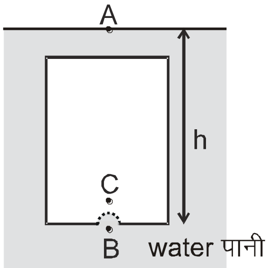
B-6. Let the container is dipped to depth h, so the contact angle becomes θ
माना पात्र को h गहराई तक डुबोया है, और स्पर्श कोण θ हो गया।

$P_A = P_0$

$P_B = P_0 + \rho g h$

$$P_C = P_0 + \rho gh - \frac{2T}{r}\cos\theta = P_0$$

$$\cos\theta = \frac{\rho ghr}{2T} \le 1$$

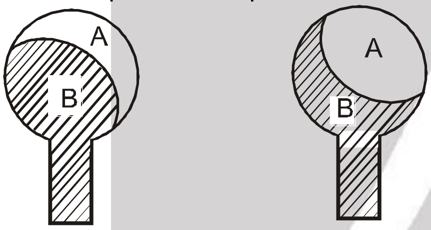


$$\boxed{h \le \frac{2T}{\rho gr}} \Rightarrow \boxed{h_{max\,\text{अधिकतम}} = \frac{2T}{\rho gr}} \quad \boxed{h_{\text{अधिकतम}} = \frac{2T}{\rho gr}}$$

PART - II

भाग - II

A-1. After the portion A is punctured’ the thread has 2 options as shown in the figures.



(i) or (ii)

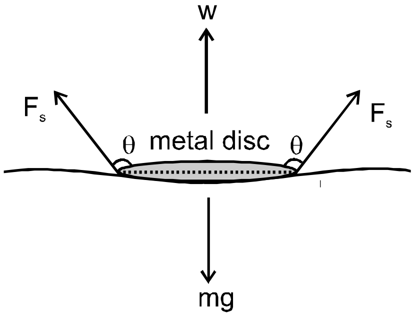
Clearly, due to surface tension , the soap film wants to minimize the surface area which is happening in option (ii).
Hence the thread will become concave towards A.
जब A भाग को पिन से तोड़ा जाता है, तो धागे के चित्रानुसार दो स्थितियाँ सम्भव हैं

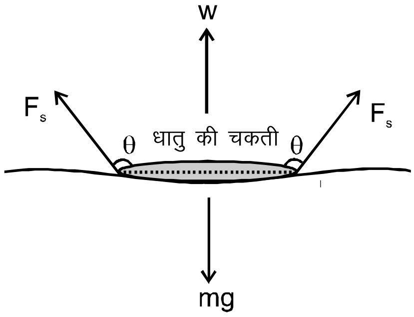
(i) या (ii)

स्पष्टतया, पृष्ठतनाव के कारण, साबुन की फिल्म पृष्ठ क्षेत्रफल को कम करने का प्रयास करेगी जो कि स्थिति (ii) में सम्भव है।
अतः धागा A की तरफ अवतल हो जायेगा।

A-2. In the satellite, g_{eff} becomes zero but the surface tension still prevails. Hence the water will experience only surface Tension force which will push it fully outward.
कृत्रिम उपग्रह में g_{eff} शून्य हो जाता है लेकिन पृष्ठ तनाव अभी भी रहता है, अतः जल पर केवल पृष्ठ तनाव बल लगेगा जो कि उसे केवल ऊपर की ओर खींचेगा।

A-3.





The FBD of disc is shown in the figure. The net upward surface tension force

$= F_S \cos\theta = (T \times 2\pi r)\cos\theta$.

so $F_S \cos\theta + W = mg = W_{disc}$

यहाँ चित्र में चकती का FBD बनाया गया है। ऊपर की ओर लगने वाला कुल पृष्ठ तनाव बल

$= F_S \cos\theta = (T \times 2\pi r)\cos\theta$.

अतः $F_S \cos\theta + W = mg = W_{disc}$

A-4. We know that surface energy

$U_S = T \times \text{Area}$.

Here. as 2 films are formed because of ring. so

$U_S = T \times 2 \times (A) = 5\ \frac{N}{m} \times 2 \times 0.02\ m^2 = 0.2\ J$

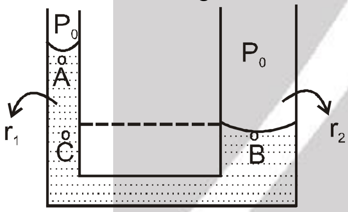
हम जानते है कि पृष्ठ ऊर्जा

$U_S = T \times$ क्षेत्रफल

यहाँ, क्योंकि वलय के कारण 2 फिल्म बनती है अतः

$U_S = T \times 2 \times (A) = 5\ \frac{N}{m} \times 2 \times 0.02\ m^2 = 0.2\ J$

A-5. In the shown diagram.



$P_C = P_B$

$P_0 - \frac{2T}{r_1} + \rho gh = P_0 - \frac{2T}{r_2}$

Here, we may not know in advance which tube will rise above the other, but lets say the liquid level is higher in thinner tube.

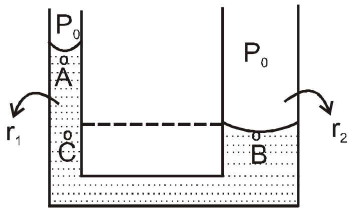
so $\quad 2T\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -\rho gh. \quad \Rightarrow \quad T = \frac{\rho gh\ r_1\ r_2}{2\ (r_2 - r_1)}$

as $\quad r_2 > r_1$; so we assumed correctly

Sol. दिये गये चित्र से

$P_C = P_B$

$P_0 - \frac{2T}{r_1} + \rho gh = P_0 - \frac{2T}{r_2}$



यहाँ हमें पहले से नही पता है कि कौनसी नली में द्रव स्तर, दूसरे की तुलना में ऊपर जायेगा लेकिन माना पतली ट्यूब (नली) में द्रव का तल ज्यादा है।

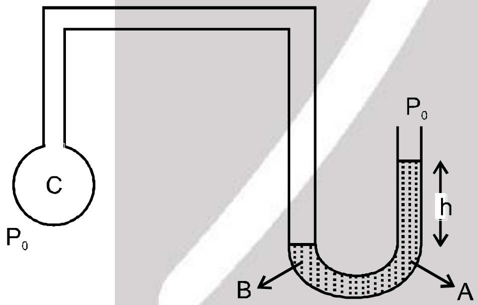
इसलिए $2T\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -\rho gh.$ $\Rightarrow$ $T = \frac{\rho gh}{2} \frac{r_1 \; r_2}{(r_2 - r_1)}$

क्योंकि $r_2 > r_1$; अतः हमने सही माना है।

A-6. Water will rise to a height more than h when downward force (mg_{eff}) becomes lesser than mg. so in a lift accelerating downwards, g_{eff} is $(g - a_0)$. Hence capillary rise is more.
On the poles g_{eff} is even more than g. Hence the capillary will even drop.
जब नीचे की तरफ लगने वाला बल (mg_{eff}), mg से कम होता है तो जल में उँचाई से ज्यादा ऊँचाई तक चढता है अत, नीचे की तरफ त्वरित लिफ्ट में g_{eff} , $(g - a_0)$ है। अतः केशनली में द्रव ज्यादा ऊपर जायेगा धुवो पर g_{eff} , g से ज्यादा होता है अतः केशनली में द्रव नीचे गिरेगा।

A-7. Insects use the surface tension force to keep floating.
कीट तैरने के लिये पृष्ठ तनाव का उपयोग करते है।

A-8.



P_A has to be equal to P_B.
P_A ,P_B के समान होना है।

$P_A = P_0 + \rho gh$ .....(i)

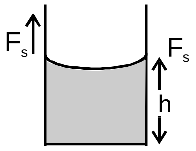
Now अब $P_C - P_0 = \frac{4\sigma}{r}$ $\because$ soap bubble has 2 films साबुन के बुलबुले में 2 फिल्म है।

and और $P_C = P_B$ $\because$ same air is filled समान वायु भरी जायेगी।

$\Rightarrow$ $P_0 + \frac{4\sigma}{r} = P_0 + \rho gh$ ....(ii)

get अतः $\sigma = \frac{\rho ghr}{4}$

A-9.

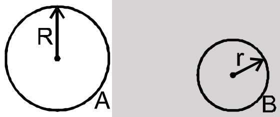


By balancing forces बल सन्तुलन से $T \times (2\ell) \times (\cos\theta) = d \times \ell h g$

we get हम प्राप्त करेगें $h = \frac{2T\cos\theta}{xdg}$

B-1. When charge is given to a soap bubble (whether positive or negative), these charges experience repulsive forces due to the other charges. Hence they tend to move out. Hence the size of bubble increases.
जब साबुन के बुलबुले को आवेश दिया जाता है (चाहे धनात्मक या ऋणात्मक) तो यह आवेश अन्य आवेशो के कारण प्रतिकर्षण बल अनुभव करेगा। अतः यह बाहर जाने की प्रवृत्ति रखेगा। अतः बुलबुले का आकार बढेगा।

B-2.



By equating volume आयतन की तुलना करने पर : $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

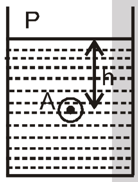
get प्राप्त होगा r = R/2.

Now pressure difference in A = $\frac{4\sigma}{R}$ अब A में दाबान्तर = $\frac{4\sigma}{R}$

and that in B = $\frac{4\sigma}{R/2}$ = 2 × pressure difference in A.

और B में दाबान्तर = $\frac{4\sigma}{R/2}$ = 2 × A में दाबान्तर

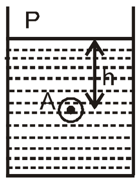
B-3.



$P_{inside\ bubble} - P_A = \frac{2T}{r}$

and $P_A = P_{atm} + dgh$.

$\Rightarrow$ $P_{inside\ bubble} = P + dgh + \frac{2T}{r}$



$P_{\text{आन्तिरक बुलबुले}} - P_A = \frac{2T}{r}$

और $P_A = P_{atm} + dgh$.

$\Rightarrow$ $P_{\text{आन्तरिक बुलबुला}} = P + dgh + \frac{2T}{r}$

B-4. $n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ ......(i){∵ volumes are equal आयतन समान है}.

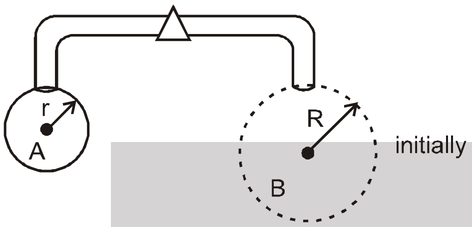
and और $\Delta A = -[4\pi R^2 - n.4\pi r^2]$

where जहां $W = (\Delta A) \times T$.

$= -4\pi[n^{2/3}r^2 - n.r^2] \times T = 4\pi r^2 T.\ n^{2/3}\ [n^{1/3} - 1]$.

Now अब $R^2 = n^{2/3}\ .\ r^2$; so इसलिए $W = 4\pi R^2 T[n^{1/3} - 1]$.

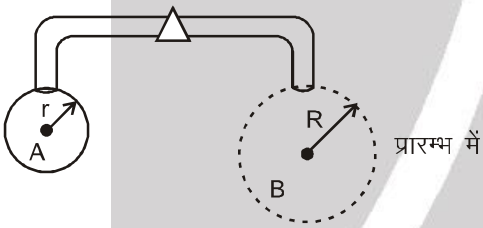
B-5.



$P_A = P_0 + \frac{4\sigma}{r}$; $P_B = P_0 + \frac{4\sigma}{R}$ {P_0 = atmospheric pressure}.

Clearly $P_A > P_B$; so air will flow from A to B.
As r decreases; pressure will become more and hence more flow of air from A to B.
Ultimately bubble A collapses and B becomes bigger in size.

हल.



$P_A = P_0 + \frac{4\sigma}{r}$; $P_B = P_0 + \frac{4\sigma}{r}$ {P_0 = वायुमण्डलीय दाब}.

स्पष्टता $P_A > P_B$; अतः वायु A से B की ओर प्रवाहित होगी।
क्योकि r कम होती है अतः दाब ज्यादा होगी ओर A से B की ओर वायु ज्यादा प्रवाहित होगी।
अतः बुलबुला A टूट जायेगा और B का आकार बढ जायेगा।

B-6.

R = 4 cm.
r = 3 cm.

$P_r = \frac{4\sigma}{r}$; $P_R = \frac{4\sigma}{R}$ { ∵ outside is vacuum बाहर निर्वात है}

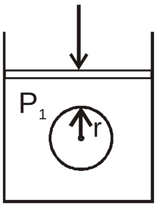
The two bubbles are coalescing; so conserving the no. the moles.
जब दोनो बुलबुलो मिलाया जाता है तब मोल सरंक्षण से

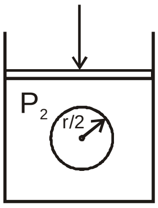
$$\frac{P_r.\frac{4}{3}\pi r^3}{T} + \frac{P_R.\frac{4}{3}\pi R^3}{T} = \frac{P_{final} \times \frac{4}{3}\pi (r')^3}{T}$$

Putting मान रखने पर $P_{final} = \frac{4\sigma}{r'}$ we get हम प्राप्त करेगे

$r' = \sqrt{r^2 + R^2} = \sqrt{3^2 + 4^2} = 5$ cm.

B-7.

Before 

After 

Lets say, initially, the pressure due to air inside the bubble is P_{air}.

$\Rightarrow \quad P_{air} - P_1 = \frac{4\sigma}{r}$ ....(i)

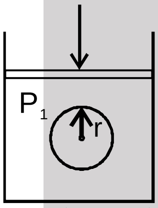
Finally, the radius becomes half ; so volume becomes $\frac{4\sigma}{r}$ th and hence pressure becomes $8P_{air}$.

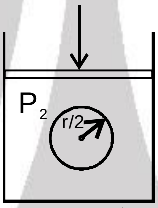
So, $\quad 8P_{air} - P_2 = \frac{4\sigma}{r/2}$ ....(ii)

Solving (i) and (ii)

get $\quad P_2 = 8P_1 + \frac{24\sigma}{r}$.

हल.

i gy s 

ckn esa 

माना प्रारम्भ में वायु के कारण बुलबुले के अन्दर दाब P_{air} है।

$\Rightarrow \quad P_{air} - P_1 = \frac{4\sigma}{r}$ ....(i)

अन्त में त्रिज्या आधी हो जायेगी अतः आयतन $\frac{1}{8}$ हो जायेगा अतः दाब $8P_{air}$ हो जायेगा।

अतः $\quad 8P_{\text{वायु}} - P_2 = \frac{4\sigma}{r/2}$ ....(ii)

(i) और (ii) को हल करने पर

प्राप्त होगा। $\quad P_2 = 8P_1 + \frac{24\sigma}{r}$.

B-8. When the excess pressure at the hole becomes equal to the pressure of water height ;then only water will start coming out of the holes : [atm pressure on both sides is same].
जब छेद पर आधिक्य दाब, जल की ऊँचाई के दाब के बराबर हो जायेगा तब केवल जल छेद से बाहर आना प्रारम्भ करेगा। [दोनो तरफ वायुमण्डलीय दाब समान होगा]

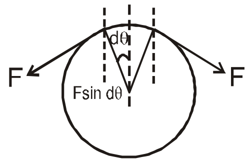
$\Rightarrow \quad \rho hg = \frac{2\sigma}{r} \quad \Rightarrow \quad h = \frac{2\sigma}{\rho rg}$

$$= \frac{2 \times 70 \times 10^{-3} \times \frac{N}{m}}{1000\frac{kg}{m^3} \times \left(\frac{0.1}{2}\right) \times 10^{-3} \times 10} = 0.28 \text{ m.}$$

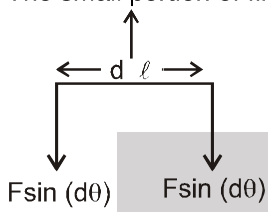
EXERCISE-2

PART - I

1.



The small portion of film is approximately a straight part. Balancing forces on it:



F denotes tension.   T denotes surface tension.

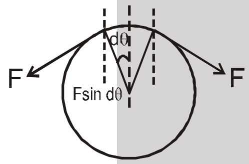
T × 2 (dℓ) is the surface tension force because 2 layers are formed.

So 2 F sin (dθ) = T × [2 × R (2 dθ)]

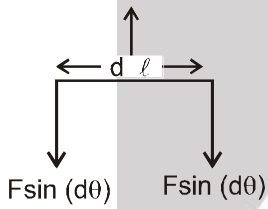
we get ;   (sin (dθ) ≈ dθ. for small dθ)

so F = T × 2 R.

Hindi.



फिल्म का छोटा भाग लगभग सीधी रेखा है तथा इस पर बलों के सन्तुलन से



F तनाव को प्रदर्शित करता है   T, पृष्ठ तनाव को प्रदर्शित करता है

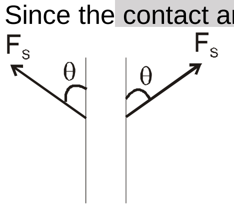
T × 2 (dℓ) बनी हुई दो सतहों के कारण पृष्ठ तनाव के कारण।

अतः 2 F sin (dθ) = T × [2 × R (2 dθ)]

छोटे कोण के लिये dθ,   sin (dθ) ≈ dθ.

अतः F = T × 2 R.

2. Since the contact angle in both cases remains the same.



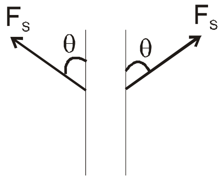
$F_S \cos\theta = Mg \Rightarrow T \times 2\pi R \cos\theta = Mg$ ....(i)

after doubling the radius

$T \times 2\pi (2R) \cos\theta = M'g$ ....(ii)

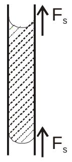
= M' = 2M.

Sol. ∴ दोनों स्थितियों में सम्पर्क कोण समान है



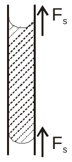
$F_S \cos\theta = Mg \Rightarrow T \times 2\pi R \cos\theta = Mg$ ......(i)
त्रिज्याओ को दुगना करने पर
$T \times 2\pi (2R) \cos\theta = M'g$.......(ii)
$= M' = 2M$.

3. When the capillary rise is 'h' that means the force of surface tension (F) is supporting the height 'h' of liquid level.
Now if the whole capillary is taken out the liquid tries to come out due to gravity from the bottom point.



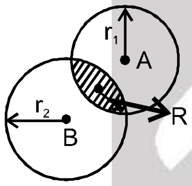
But force of surface tension 'F' now becomes 2F in the upward direction. Hence 2F can support a maximum of '2h' height even if ℓ is very high. So 'h' will be 2h if $\ell > h$ & will be $h + \ell$ only if ℓ is lesser than h.

हल. जब केशनली मे जल 'h' ऊचांई तक ऊपर उठती है अर्थात पृष्ठ तनाव द्रव सतह जो 'h' ऊचाई तक सहारा देता है।
अब यदि केशनली से जल बाहर निकलता है तो द्रव गूरूत्व के कारण निम्न बिन्दु से बाहर निकलता है।



लेकिन अब पृष्ठ तनाव 'F' ऊपर की ओर 2F बन जायेगा। अतः 2F अधिकतम '2h' ऊचाई तक सहारा देगा यद्यपि ℓ बहुत बडा है। अतः 'h' 2h होगा यदि $\ell > h$ और, यदि ℓ, h से कम है तो h' $(h + \ell)$ के बराबर होगा।

4.

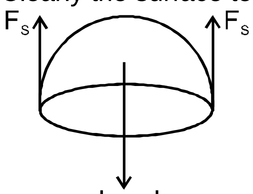


Equating pressures on the shaded portion : छायाकित भाग का दाब बराबर करने पर

$$\frac{4\sigma}{r_1} - \frac{4\sigma}{r_2} = \frac{4\sigma}{R}$$

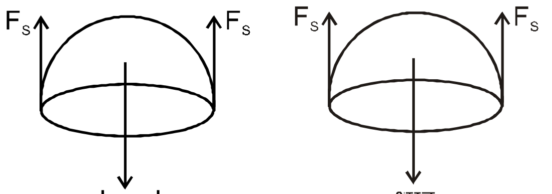
get प्राप्त होगा $R = \frac{r_2 r_1}{r_2 - r_1}$

5. Clearly the surface tension force on



Hemisphere = $F_S = (2T).(2\pi r)$ {2 layers are formed}.
$\Rightarrow F_S = 2 \times 500$ N/m $\times 2 \times 3.14 \times 5$m $\approx$ 30,000 N $\approx$ 3000 kg.wt.

हल. स्पष्टतया अर्द्वगोले पर पृष्ठ तनाव बल

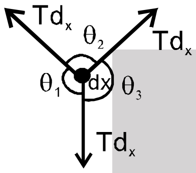


अर्द्वगोले पर पृष्ठ तनाव बल $= F_S = (2T).(2\pi r)$ {2 सतह बनेगी}.

$\Rightarrow \quad F_S = 2 \times 500 N/m \times 2 \times 3.14 \times 5m \approx 30,000\ N \approx 3000$ kg.wt.

6. Look at a very small element at the junction of 3 bubbles.

तीनो बुलबुलो के मिलान बिन्दु पर एक बहुत छोटे अवयव को देखते है।



All 3 forces of same magnitude (surface tension is same) are acting along the tangential directions on the small element.

समान परिमाण के तीनो बल (पृष्ठतनाव समान है) छोटे आवयव पर त्रिभुजीय दिशा में कार्य करेगे।

Now by LAMIE's theorem अब लामी प्रमेय से

$\theta_1 = \theta_2 = \theta_3 = \dfrac{360}{3} = 120^o$

7. Energy released = $(\Delta A) \times \sigma$ {σ = surface tension}

उत्सर्जित ऊर्जा = $(\Delta A) \times \sigma$ {σ = पृष्ठ तनाव }

Let us say n no. of small drops coalesced.

माना छोटी बूदे जो मिलती है उनकी संख्या n है।

$\Rightarrow \quad n.\ \dfrac{4}{3}\pi a^3 = \dfrac{4}{3}\pi b^3 \ \Rightarrow\ b = a.n^{1/3} \ \Rightarrow\ n = \left(\dfrac{b}{a}\right)^3$

$\Delta A = 4\pi b^2 - n.4\pi a^2$ {this is –ve, hence energy is released} {यह ऋणात्मक है अतः ऊर्जा उत्सर्जित होगी}

$= 4\pi a^2 (n^{2/3} - n)$

$\Rightarrow \quad U = 4\pi a^2 T\ (n - n^{2/3}) = 4\pi a^2 T \left[\left(\dfrac{b}{a}\right)^3 - \left(\dfrac{b}{a}\right)^2\right]$

This U converts to K.E. यह स्थितिज ऊर्जा गतिज ऊर्जा में परिवर्तीत हो जायेगी।

Hence अतः $\dfrac{1}{2}\rho.\dfrac{4}{3}\pi b^3\ \ V^2 = 4\pi a^2 T\ \dfrac{b^2}{a^2}\left(\dfrac{b-a}{a}\right) \ \Rightarrow\ V = \sqrt{\dfrac{6T}{\rho}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)}$

8. Surface tension is a property based on intermolecular force, at critical temperature intermolecular force is zero, hence surface tension is zero.

पृष्ठ तनाव अन्तर आणविक बल पर आधारित गुण है, क्रांतिक ताप पर अन्तर आणविक बल शून्य है, अतः पृष्ठ तनाव शून्य है।

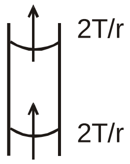
9. $\dfrac{4T}{R} = \rho g(2mm)$

$\Rightarrow \quad \dfrac{8T}{d} = \rho g\ (2\ mm)$

$\Rightarrow \quad T = \dfrac{0.8 \times 980 \times 3 \times (.2)}{8} = 9.8 \times 6 = 58.8$

PART - II

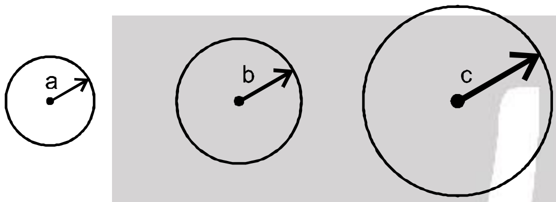
2. $\frac{4T}{r} = h\rho g$



$\frac{4 \times 72}{0.1} = h \times 1 \times 1000$

h = 2.88 cm

3.



Let (a) and (b) coalesce to form (c). माना (a) व (b) मिलकर (c) बनाते है।

By mole conservation : अतः मोल सरक्षण से

$P_a . a^3 + P_b . b^3 = P_c . c^3$ .... (i)

Also और $P_a = P_0 + \frac{4\gamma}{a}$ ....(ii)

$P_b = P_0 + \frac{4\gamma}{b}$ ....(iii)

$P_c = P_0 + \frac{4\gamma}{c}$ ....(iv)

Putting there values : प्रतिस्थापन करने पर

$$\left(P_0 + \frac{4\gamma}{a}\right) a^3 + \left(P_0 + \frac{4\gamma}{b}\right) b^3 = \left(P_0 + \frac{4\gamma}{c}\right) c^3$$

$$P_0 \left[a^3 + b^3 - c^3\right] + 4\gamma \left[a^2 + b^2 - c^2\right] = 0$$

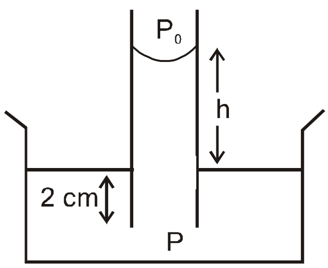
$\Rightarrow$ also तथा $c^3 - (b^3 + a^3) = \frac{3V}{4\pi}$ and और $c^2 - (a^2 + b^2) = \frac{s}{4\pi}$

Putting there values : मानो का प्रतिस्थापन करने पर

$$P_0 \left(\frac{-3v}{4\pi}\right) + 4T \left(\frac{-S}{4\pi}\right) = 0$$

$\Rightarrow$ $3P_0V + 4ST = 0$ **Ans. (A).**

4.



Assuming the contact angle to be 0º.

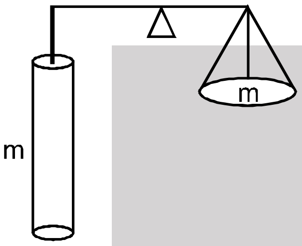
$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.075 \frac{N}{m} \times 1}{0.5 \times 10^{-3} \times 1000 \frac{kg}{m^3} \times 9.8} = 3 \text{ cm.}$$

Now the pressure at point P (figure); 2 cm. below the water surface effectively will be.
$\rho g \times 2$ cm = 196 N/m^2.
स्पर्श कोण को 0^o मान कर

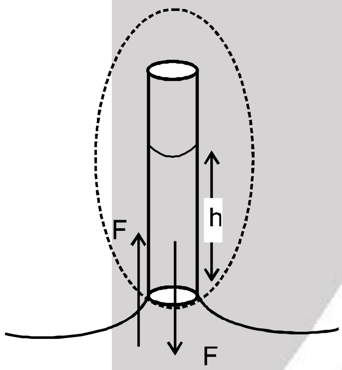
$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.075 \frac{N}{m} \times 1}{0.5 \times 10^{-3} \times 1000 \frac{kg}{m^3} \times 9.8} = 3 \text{ cm.}$$

अब पानी की सतह से 2 cm. नीचे बिन्दु P पर दाब होगा।
$\rho g \times 2$ cm = 196 N/m^2.

5. Let the mass of capillary tube only is 'm' so, the balanced beam is given by.
मान केवल केशनली का द्रव्यमान 'm' है, अतः भुजा की सन्तुलन की अवस्था में

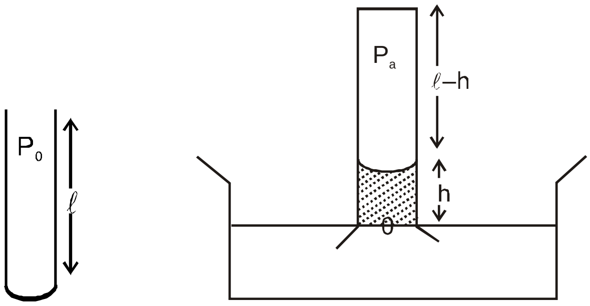


Now the end of capillary is brought in contact with the water,. so some height is raised in the capillary.
अब केशनली का सिरा जल के सम्पर्क में लाया जाता है, अतः केशनली में ऊँचाई बढ़ जाती है



Lets say, our system is capillary plus the water inside of height 'h'
The force of surface tension at the end is being felt by capillary in the downward direction.
And the force of surface tension at the top becomes internal force for are system. so.
F + ρhg A = [extra mass (0.135 gm)] × g.
and we know for capillary risen water $\rho gh \pi r^2 = T \times 2\pi r$.
so $2T \times 2\pi r = (0.135 \times 10^{-3}) \times g$
get $r \approx 1.5$ mm.
हम केशनली तथा 'h' ऊँचाई तक के पानी को अपना निकाय मान लेते है। केशनली के द्वारा सिरे पर पृष्ठतनाव बल निचे की तरफ महसूस किया जायेगा। तथा पृष्ठतनाव बल शिखर पर आन्तरिक बल बन जाता है। इसलिये .
F + ρhg A = [अतिरिक्त द्रव्यमान (0.135 gm)] × g.
तथा केशनली की ऊँचाई में बढोतरी के लिये हम जानते है कि $\rho gh \pi r^2 = T \times 2\pi r$.
इसलिय $2T \times 2\pi r = (0.135 \times 10^{-3}) \times g$
प्राप्त करते है $r \approx 1.5$ mm.

6. माना केश नली की लम्बाई = ℓ.



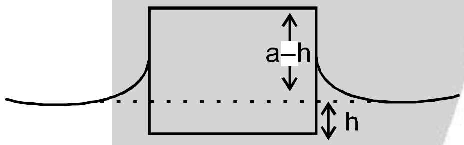
Now अब $P_a \times (\ell - h) = P_0 \times \ell$ ....(1) (isothermal condition)

& $P_a - \frac{2\sigma}{r} + \rho gh = P_0$

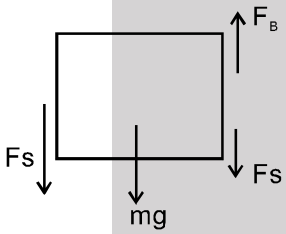
$\Rightarrow \frac{P_0\ell}{\ell - h} - P_0 = \frac{2\sigma}{r} - \rho gh$ $\Rightarrow \frac{P_0}{\ell - h} = \frac{2\sigma}{rh} - \rho g$

$\Rightarrow \ell - h = \frac{P_0 \; rh}{2\sigma - \rho grh}$ $\Rightarrow \ell = h + \frac{P_0 \; rh}{2\sigma - \rho grh} \approx 556.55$ cm

7.



FDB of cube नली का मुक्त वस्तु चित्र



F_B = buoyant force F_s = surface tension force
F_B = उत्पलावन बल F_s = पृष्ठ तनाव बल

$\Rightarrow -(4a) \times \alpha + d \times g \times a^2h = mg.$ $\Rightarrow h = \frac{mg + 4a\alpha}{dga^2} = 2.3$ cm

8. Here, the work done by surface tension force is being converted into gravitational potential energy and heat.

So $W_{Fs} = U_g$ + heat

$\Rightarrow (2\pi r)(T) \times (h) = mg\, h/2$ + heat {h/2 because of P. E. of com.}

$\Rightarrow 2\pi T \times r \times \frac{2T}{r\rho g} = \frac{(\rho g \times \pi r^2 \times h) \times 2T}{r\rho g} \times \frac{1}{2}$ + heat

get heat evolved = $\frac{2\pi T^2}{\rho g}$

Sol. यहाँ पृष्ठ तनाव बल द्वारा किया गया कार्य गुरूत्वीय स्थितिज ऊर्जा तथा ऊष्मा में परिवर्तित किया जा रहा है।

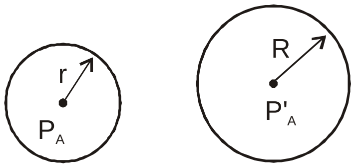
इसलिए $W_{Fs} = U_g$ + ताप

$\Rightarrow (2\pi r)(T) \times (h) = mg\, h/2$ + ऊष्मा {h/2 स्तम्भ की P. E. के कारण}

$\Rightarrow 2\pi T \times r \times \frac{2T}{r\rho g} = \frac{(\rho g \times \pi r^2 \times h) \times 2T}{r\rho g} \times \frac{1}{2}$ + heat ऊष्मा

उत्पन्न ऊष्मा = $\frac{2\pi T^2}{\rho g}$ है।

9. we know हम जानते है $\frac{K.Q}{R} = V \quad Q = \frac{VR}{K}$ $\quad \sigma = \frac{VR}{4\pi R^2 K} = \frac{V\varepsilon_0}{R}$



[$\frac{\sigma^2}{2\varepsilon_0}$ is excess pressure due to uniform charge distribution on the surface of a bubble pressure is larger than outside]

[$\frac{\sigma^2}{2\varepsilon_0}$ अतिरिक्त दाब है जो कि बुलबुले की सतह पर एक समान आवेश वितरण से उत्पन्न हुआ है। अन्दर का दाब बाहर के दाब से अधिक होगा।]

Clearly स्पष्टतः $P_A \times \frac{4}{3}\pi r^3 = P_A' \times \frac{4}{3}\pi R^3$ $\quad \Rightarrow P_A = P_A\left(\frac{r}{R}\right)^3$

Now अब $P_A - P_0 = \frac{4T}{r}$ & $P_A - P_0 = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$ ......(3)

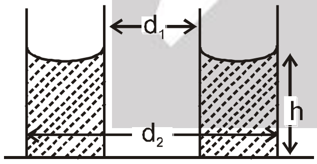
so from अतः (3) से $P_A\left(\frac{r}{R}\right)^3 - P_0 = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$ ......(4)

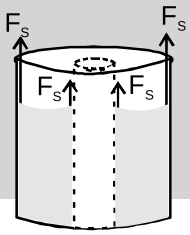
$$\left[Eq^n(2)\times\left(\frac{r}{R}\right)^3 - Eq^n(4)\right]$$

$P_0 - P_0\left(\frac{r}{R}\right)^3 = \frac{4T}{r}\left\{\left(\frac{r}{R}\right)^3 - \frac{r}{R}\right\} + \frac{V^2\ \varepsilon_0^2}{2R^2\varepsilon_0}$ $\quad \Rightarrow P_0(R^3 - r^3) = \frac{4T}{r}\{r^3 - rR^2\} + \frac{V^2\varepsilon_0 R}{2}$

$\Rightarrow P_0(R^3 - r^3) + 4T(R^2 - r^2) - \frac{V^2\varepsilon_0 R}{2} = 0.$ Hence provide अतः सिद्ध हुआ **Ans.** $\lambda = 4$

10. Now there will be two meniscuses. यहाँ दो नवचन्द्रक (meniscuses) होगें
Net upward surface tension force is balanced by the weight of the column.
कुल पृष्ठ तनाव बल स्तम्भ के भार से सुतुलित है।





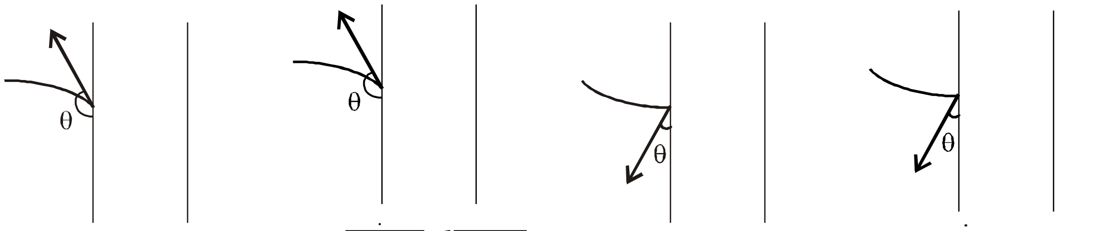
$\Rightarrow T \times 2\pi \frac{d_1}{2} + T \times 2\pi \frac{d_2}{2} = \pi \times \left(\frac{d_2^2 - d_1^2}{4}\right) \times h\rho g.$

we get प्राप्त करते है $h = \frac{4T}{\rho g(d_2 - d_1)} = 6$ cm.

11. $F = 2T\ell = 2 \times 40 \times 6 = 480$ dyne
$W = F x = 480 \times 0.2 = 96$ erg

PART - III

10. Force of cohesion keeps the molecules of a material bounded together and does not let them stick to the solid as force of adhesion is lesser.
ससंजक बल किसी प्रदार्थ के अणुओ को एक दूसरे से बाधे रखते है तथा क्योकि आंसजक बल कम है अतः इन्हे ठोस से जुडने नही देता है।



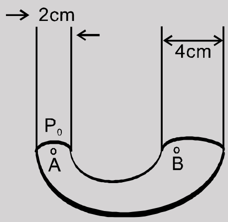
Cohesion more than adhesion | ससंजक, आसजक से ज्यादा होगा | adhesion more than cohesion | आसंजक, ससजक से ज्यादा होगा

Hence अतः (A) (B) (C).

PART - IV

1 & 2. Pressure difference = $\frac{2T}{r_1} - \frac{2T}{r_2}$ {∵angle of contact is 180º}

दाब में अन्तर = $\frac{2T}{r_1} - \frac{2T}{r_2}$ {∵ स्पर्श कोण 180º है}



$\Rightarrow \quad \rho_{Hg}\, h_{Hg} \times g = 2T\left(\frac{r_2 - r_1}{r_1 \;\; r_2}\right)$

$\Rightarrow \quad h_{Hg} = \frac{2T}{\rho g}\left(\frac{r_2 - r_1}{r_1 \;\; r_2}\right) = 3.53$ mm of Hg.

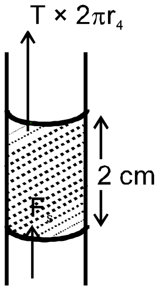
3 to 5. As $P_A > P_B$, although they are at same height, hence the air above the point B has been evacuated. So the bigger limb of the tube should be connected to the pump.
जैसे कि $P_A > P_B$ वैसे तो वे समान ऊँचाई पर है, अतः वायु को बिन्दु B से हटा दिया गया है इसलिए नली के बड़े लिम्ब (limb) को पम्प से जोड़ा चाहिए।
As the wetting is complete. so upper meniscus radius will be same as radius of capillary in all cases.

$r_4 = \frac{r_{capillary}}{2} = 0.5$ mm.

क्योंकि सम्पूर्ण जगह पर गीलापन है अतः ऊपरी नवचन्द्रक की त्रिज्या सभी स्थितियों में केशनली की त्रिज्या के बराबर होगी।

$r_4 = \frac{r_{\text{केशनली}}}{2} = 0.5$ mm.

Now अब (a) $F_S = T \times 2\pi rl$.
so balancing forces in vertical direction. इसलिए ऊर्ध्व दिशा में बलों को सन्तुलित करने पर



$\rho \times (\pi r_c^2 \times h) \times g = T \times 2\pi r_l + T \times 2\pi r_c$.

get प्राप्त करते है $r_l = \frac{\rho h g r_c^2}{2T} - r_c$

EXERCISE-3

PART - I

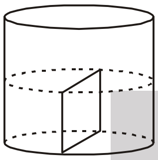
भाग - I

1. Pushing force दबाव बल = <p> (Area) (क्षैत्रफल)

$= \left(\frac{(p_0) + (p_0 + \rho gh)}{2}\right)$ (2Rh)

$= 2p_0 Rh + \rho g h^2 R$

Pulling force खींचाव बल = (T) (2R)

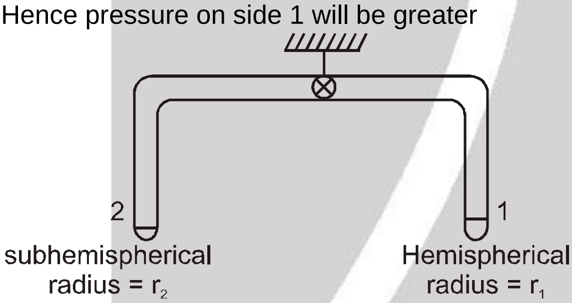


Net force परिणामी बल = $\left| 2p_0Rh + \rho gh^2R - 2TR \right|$

2. Pressure inside tube = $P = P_0 + \frac{4T}{r}$

$\therefore P_2 < P_1$ (since $r_2 > r_1$)

Hence pressure on side 1 will be greater



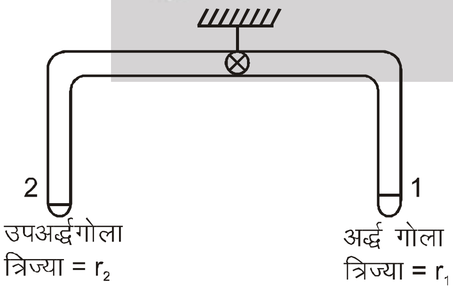
than side 2. So air from end 1 flows towards end 2.

Ans. (B)

Sol. ट्यूब के अन्दर दाब = $P = P_0 + \frac{4T}{r}$

$\therefore P_2 < P_1$ (since $r_2 > r_1$)

अतः 1 तरफ की भुजा में दाब भुजा 2 से अधिक होगा अतः उत्तर (B) है।



सिरे 1 से सिरे 2 की तरफ हवा का बहाव होगा।

Ans. (B)

3. $P_A = P_0 + \frac{4T}{r_A} \Rightarrow P_A = 8 + \frac{4 \times 0.04}{0.02}$

$P_A = 16\ N/m^2$

$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04}$

$P_B = 12\ N/m^2$

for bubble A, PV = nRT

$(16)\ \frac{4}{3}\pi\ (0.02)^3 = n_A\ RT$ ....(1)

for bubble B

$(12)\ \left(\frac{4}{3}\pi(0.04)^3\right) = n_B RT$ ....(2)

dividing eq^n (i) and (2) $\frac{n_A}{n_B} = \frac{1}{6}$; $\frac{n_B}{n_A} = 6$ **Ans. 6**

Sol. $P_A = P_0 + \frac{4T}{r_A} \Rightarrow P_A = 8 + \frac{4 \times 0.04}{0.02}$

$P_A = 16\ N/m^2$

$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04}$

$P_B = 12\ N/m^2$

बुलबुले A के लिए, PV = nRT

$(16)\ \frac{4}{3}\pi\ (0.02)^3 = n_A\ RT$ ....(1)

बुलबुले B के लिए

$(12)\ \left(\frac{4}{3}\pi(0.04)^3\right) = n_B RT$ ....(2)

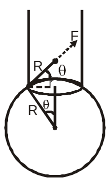
समीकरण (1) में (2) का भाग देने पर

$\frac{n_A}{n_B} = \frac{1}{6}$

$\frac{n_B}{n_A} = 6$

Ans. 6

4.



Due to surface tension, vertical force on drop = $F_v = T2\pi r\ \sin\theta = T2\pi r \frac{r}{R} = \frac{T2\pi r^2}{R}$

पृष्ठ तनाव के कारण बूँद पर ऊर्ध्वाधर बल = $F_v = T2\pi r\ \sin\theta = T2\pi r \frac{r}{R} = \frac{T2\pi r^2}{R}$

5. Equating forces on the drop :
बूँद पर बलों का सन्तुलन करने पर

$$\frac{T2\pi r^2}{R} = \rho\frac{4}{3}\pi R^3 g$$

(Assume drop as a complete sphere)
(बूँद को एक गोला मानें)

$$R = \left(\frac{3Tr^2}{2\rho g}\right)^{1/4} = \left(\frac{3\times 0.11\times 25\times 10^{-8}}{2\times 10^3\times 10}\right)^{1/4}$$

$= 14.25 \times 10^{-4}$ m $= 1.425 \times 10^{-3}$ m

6. Surface energy of the drop
बूँन्द की पृष्ठ ऊर्जा

$U = TA$
$= 0.11 \times 4\pi\,(1.4 \times 10^{-3})^2$
$= 2.7 \times 10^{-6}$ J

7. Surface Tension $\gamma = \dfrac{\text{force}}{\text{length}}$

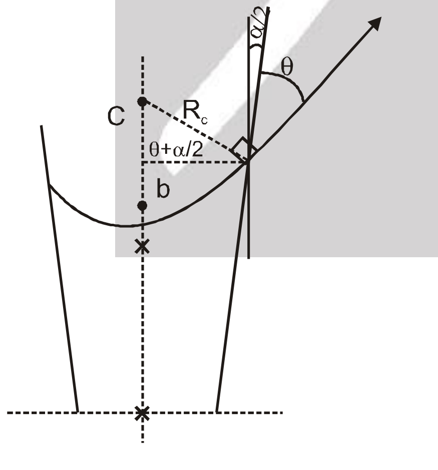
$$2\left[\frac{\sqrt{2}kq^2}{a^2} + \frac{kq^2}{2a^2}\right] = \gamma \times a\sqrt{2} \times 2$$

$a = (\text{Some constant})\left(\dfrac{q^2}{\gamma}\right)^{\frac{1}{3}}$ So N = 3 **Ans.**

8. Using geometry ज्यामिति के उपयोग से : $\dfrac{b}{R_e} = \cos\left(\theta + \dfrac{\alpha}{2}\right)$

Using Pressure method दाब विधि से : $P_0 - \dfrac{2S}{R_c} + h\rho g = P_0$

$$\Rightarrow h = \frac{2S}{R_e\rho g} = \frac{2S}{b\rho g}\cos\left(\theta + \alpha/2\right)$$



9. $R = K^{1/3}r$

$\Delta U = S.K.4\pi r^2 - S.4\pi R^2$

$$\Delta U = 4\pi S\left[K.\frac{R^2}{K^{2/3}} - R^2\right]$$

$= 0.1 \times 10^{-4}[K^{1/3} - 1] = 10^{-3}$
$K^{1/3} - 1 = 10^2$
$K^{1/3} = 101 = (10^{\alpha})^{1/3}$ $\alpha = 6$

10. $h = \dfrac{2\sigma\cos\theta}{\rho g r}$

When lift is going up with constant acceleration जब लिफ्ट ऊपर कि तरफ नियत त्वरण से गतिशील है

$h = \dfrac{2\sigma\cos\theta}{\rho(g+a)r}$

PART - II

भाग - II

2. $W = T\Delta A$

$= 0.03\,(2 \times 4\pi \times (5^2 - 3^2)\,10^{-4} = 24\pi\,(16) \times 10^{-6}$

$= 0.384\,\pi \times 10^{-3}$ Joule $0.4\,\pi$ mJ **Ans.**

3. $2.\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$R = 2^{1/3}\,r$

S.E. $= T.4\pi R^2$

$T\,4\pi\,2^{2/3}\,r^2 \qquad T.2^{8/3}\,\pi r^2$.

4. $2TL = mg$

$T = \dfrac{mg}{2L} = \dfrac{1.5\times10^{-2}}{2\times30\times10^{-2}} = \dfrac{1.5}{600} = 0.025$ N/m

5. When radius is decrease by dr

decrease in surface energy = Heat required for vaporisation

$(4\pi r dr) \times T \times 2 = 4\pi r^2 dr L\,\rho$

$\Rightarrow r = \dfrac{2T}{\rho L}$

Ans. (4)

जब त्रिज्या dr से कम हो जाती है तब

पृष्ठ ऊर्जा में कमी = वाष्पन के लिए आवश्यक ऊष्मा

$(4\pi r dr) \times T \times 2 = 4\pi r^2 dr L\,\rho$

$\Rightarrow r = \dfrac{2T}{\rho L}$

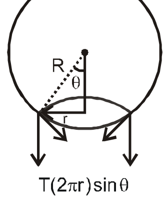
Ans. (4)

6. $\sin\theta = \dfrac{r}{R}$

The bubble will detach if -

Buoyant force $\geq$ Surface tension force

$(\rho_w)\left(\dfrac{4}{3}\pi\ \ R^3\right)\ g \geq\ (T)\,(2\pi r)\,\sin\theta$



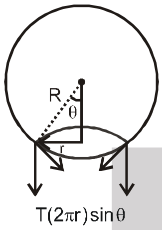
Solving $r = \sqrt{\frac{2\rho_w R^4 g}{3T}}$

No option matches with the correct Answer (BONUS).

बुलबुला अलग होगा यदि -

उत्प्लावन बल पृष्ठ तनाव के कारण

$(\rho_w)\left(\frac{4}{3}\pi \ R^3\right) \ g \geq (T)\ (2\pi r) \sin\theta$



हल करने पर $r = \sqrt{\frac{2\rho_w R^4 g}{3T}}$

कोई भी विकल्प मिलान नहीं कर रहा

HIGH LEVEL PROBLEMS (HLP)

विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

1. Weight of liquid = surface tension force

द्रव का भार = पृष्ठ तनाव बल

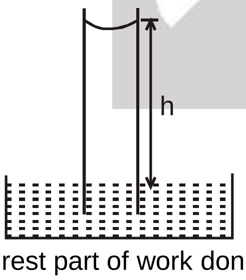
$\pi r^2 hdg = \alpha\ 2\pi r$

$h = \frac{2\alpha}{dgr}$

work done by surface tension force = $(\alpha\ .\ 2\pi r)\ h = \frac{4\pi\alpha^2}{dg}$

पृष्ठ तनाव बल द्वारा किया गया कार्य = $(\alpha\ .\ 2\pi r)\ h = \frac{4\pi\alpha^2}{dg}$

potential energy liquid द्रव की स्थितिज ऊर्जा = $mg\ h/2 = \pi r^2 h\ dg\ \frac{\alpha}{dgr} = \frac{2\pi\alpha^2}{dg}$

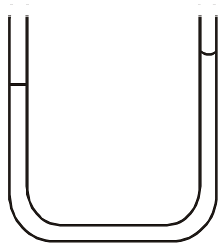


rest part of work done किये गये कार्य का शेष भाग

i.e. $\left[\frac{4\pi\alpha^2}{dg} - \frac{2\pi\alpha^2}{dg} = \frac{2\pi\alpha^2}{dg}\right]$ is lost in the form of heat. का उष्मा के रूप मे ह्रस होता है

2. $h = \frac{2T}{r_1\rho g} - \frac{2T}{r_2\rho g}$

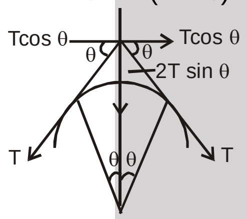
$1.25 = \frac{2T}{\rho g}\left[\frac{2}{0.1} - \frac{2}{0.2}\right]$

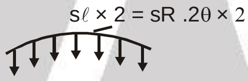


$\rho = \frac{2x49x2}{0.2x980x1.25}$

$\rho = \frac{1}{1.25}$ = 0.8 gm/cm^3

3. 2T sin θ = s (R. 2θ) x 2 for small θ sin θ = θ





छोटे θ के लिए

$T = 2SR = 2 \times .030 \times \frac{6.28x10^{-2}}{2x3.14} = 6 \times 10^{-4}$ N

4. $p_{excess} = \frac{4T}{r} = \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 20$ N/m^2

$p = p_{atm} + h\rho g + = 1.01 \times 10^5 + .40 \times 1.2 \times 10^3 \times 9.8 + \frac{2 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$

$= 1.05714 \times 10^5$ N/m^2

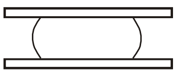
5. $\pi R^2 h = \pi R_1^2, \frac{h}{2}\ R_1 = \sqrt{2}\,R$

$mg = \rightarrow\left(\frac{2T\cos\theta}{h}\pi R^2\right)$ for balancing the plate प्लेट के संतुलन के लिए

$w + mg = \left(\frac{4T\cos\theta}{h}2\pi R^2\right)$ $\rightarrow$ for " plate + weight" "प्लेट + भार" के लिए

$w = \frac{6T\cos\theta\pi R^2}{h} = \frac{6x0.49x\cos 135°\pi(2\ \ x\ \ 10^{-2})^2}{0.38x10^{-3}}$

$= \frac{6x4.9x\cos 135°\pi x4}{38}$ N



$= \frac{12\pi\cos 135°}{38}$ kg

= 0.7 kg

6. If h is the height of water column in the capillary, the temperature of the capillary, and hence of water at this height, is
यदि केशनली में पानी स्तम्भ की ऊँचाई h है, केशनली का ताप और अतः इस ऊँचाई पर पानी

$$T_h = \frac{T_{UP}h}{\ell}$$

Water is kept in the capillary by surface tension. If σ_h is the surface tension at the temperature T_h, we can write
केशनली में पानी पृष्ठ तनाव के द्वारा रूका हुआ है। यदि T_h ताप पर पृष्ठतनाव σ_h है, हम लिख सकते है

$$h = \frac{2\sigma_h}{\rho_w gr},$$

where ρ_w is the density of water. Hence we obtain जहां ρ_w घनत्व है, अतः हम प्राप्त करते है।

$$\sigma_h = \frac{\rho grh}{2} = \left(\frac{\rho grl}{2}\right)\left(\frac{T_h}{T_{up}}\right).$$

Using the hint in the conditions of the problem, we plot the graph of the function $\sigma(T)$. The temperature T_h on the level of the maximum ascent of water is determined by the point of intersection of the curves describing the $(\rho grl/2)\ T/T_{up}$ and $\sigma(T)$ dependences.
दी गई सुचना के उपयोग से हम $\sigma(T)$ का आरेख आरेखीत कर सकते है। पानी के अधिकतम ऊँचाई के स्तर पर पानी का तापमान T_h को $\sigma(T)$ तथा $(\rho grl/2)\ T/T_{up}$ द्वारा प्रदर्शित किये जाने वाले वक्रो के प्रतिच्छेद बिन्दु से प्राप्त कर सकते है।

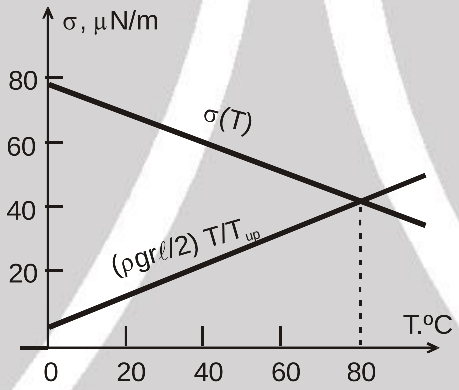
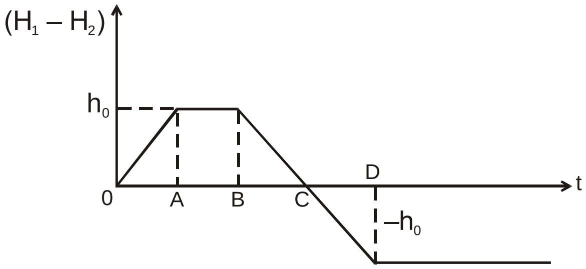


Figure shows that T_h 80ºC. Consequently, दर्शाये गये चित्र से T_h 80ºC.

$$h = \frac{\ell T_h}{T_{up}} \quad 6.4 \text{ cm.}$$

The problem can also be solved analytically if we note that the $\sigma(T)$ dependence is practically linear.
हम इस प्रश्न को विश्लेषण के द्वारा भी हल कर सकते है यदि हम ध्याने दे कि $\sigma(T)$ निर्भरता प्रायोगिक रूप से रेखीय है।

7. From the moment the filling begins to the moment of time A (figure) the water level will uniformly rise in the capillary tube (curve I) and remain at the same level in the broad tube (curve II).
भरना प्रारम्भ होने के क्षण से समय A (चित्र) के क्षण तक केशनली में (वक्र I) पानी का स्तर एक समान रूप से बढता है और चौडी नली (वक्र II) में भी एक समान रूप से बढता है।



The difference in the levels will constantly increase (figure). At the moment of time A the difference in the level will reach $h_0 = \frac{2\alpha}{gr}$

स्तरों में अन्तर लगातार बढता रहता है। A क्षण पर स्तरो में अन्तर $h_0 = \frac{2\alpha}{gr}$ पर पहुँच जाता है।

From this moment up to the moment of time B the levels in the capillary and broad tube will rise with the same velocities while the difference in the levels will remain constant and equal to h_0.

इस क्षण से B समय के क्षण तक केशनली व चौडी नली में स्तर समान वेग से बढता है जबकि स्तरो में अन्तर अपरिवर्तित रहता है व h_0 के बराबर होता है।

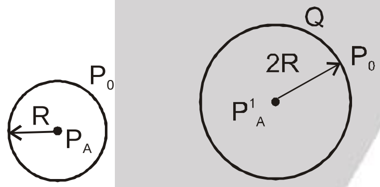
At the moment of time B the water level in the capillary tube will reach the end of the capillary and will stop at a height h_1 (figure). From the moment B to the moment D the water level will continuously rise in the broad tube. The water level in the capillary will remain constant but the meniscus will change its shape from concave of radius r (at the moment B) to a flat one (at the moment C) and then to a convex one of radius r (at the moment D). The difference in the levels in the section BC will decrease to zero and in the section CD it will change its sign and will increase to h_0.

B क्षण पर केशनली में पानी का स्तर केशनली के सिरे तक पहुँच जाता है व h_1 (चित्र) ऊँचाई पर रूक जाता है क्षण B से क्षण D तक चौडी नली में पानी का स्तर लगातार बढता हैं केशनली में पानी का स्तर वही रहता है लेकिन नवचन्द्र परिवर्तन होता रहता है इसका आकार r त्रिज्या के अवतल (क्षण B पर) से समतल (C क्षण पर) और तब r त्रिज्या का उत्तल (D क्षण पर) है BC खण्ड में स्तरो का अन्तर घटकर शून्य होता है। और खण्ड CD में इसके चिन्ह परिवर्तन होता है और h_0 तब बढता है।

At the moment D the water will begin to flow out of the capillary tube and from this moment onwards all the levels will be constant. The maximum height to which the water rises in the broad tube is $h_1 + h_2$. The maximum difference in the levels is h_0..

D क्षण पर पानी केशनली से बाहर निकलना प्रारम्भ कर देता है और इस क्षण के बाद सभी स्तर स्थिर हो जाते है। चौडी नली में पानी स्तर की अधिकतम ऊँचाई $h_1 + h_2$ है। स्तरो में अधिकतम अन्तर h_0 है।

8.



by conservation of mole : मोल संरक्षण सिद्धान्त से

$P_A \times V = P'_A \times (8V)$ {$\therefore$ doubling the radius, volume gets 8 times }

{$\therefore$ त्रिज्या को दुगुना करने पर आयतन 8 गुना हो जाता है।}

$\Rightarrow P'_A = \frac{P_A}{8}$

Now for radius R:

$P_A - P_0 = \frac{4T}{R}$

& $\frac{P_A}{8} - P_0 = \frac{4T}{2R} - \frac{\sigma^2}{2\varepsilon_0}$ {$\therefore$ pressure by charge density σ is given by $\frac{\sigma^2}{2\varepsilon_0}$ }

{$\therefore$ आवेश घनत्व σ के कारण दाब $\frac{\sigma^2}{2\varepsilon_0}$ दिया जा सकता है}

Now अब समीकरण $Eq^n\left[(1)\times\frac{1}{8}\right] - Eq^n[2]$ we get हम प्राप्त करते है $\frac{7P_0}{8} = -\frac{3T}{2R} + \frac{\sigma^2}{2\varepsilon_0}$

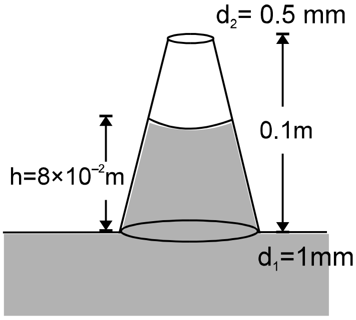
$\Rightarrow \sigma^2 = \left(\frac{7P_0}{8} + \frac{3T}{2R}\right) 2\varepsilon_0 \Rightarrow \sigma = \sqrt{\left(\frac{7P_0}{8} + \frac{3T}{2R}\right) 2\varepsilon_0}$

$\Rightarrow Q = 4\pi (2R)^2 \sqrt{\left(\frac{7P_0}{8} + \frac{3T}{2R}\right) 2\varepsilon_0} = 8\pi R \sqrt{4R^2 \left(\frac{7P_0}{8} + \frac{3T}{2R}\right) 2\varepsilon_0}$

$= 8\pi R \sqrt{\varepsilon_0 R \ (7P_0R + 12T)}$

Hence proved. अतः सिद्ध हुआ।

9.



$r_h = r_1 - \left(\frac{r_1 - r_2}{\ell}\right) L \qquad r_h = 0.5 \times 10^{-3} - \frac{(0.5-0.25)}{0.1} \times 10^{-3} \times 8 \times 10^{-2}$

$= 0.3 \times 10^{-3} m \; ; \; \frac{2T_0}{r_h} = \rho g h \Rightarrow T_0 = \frac{\frac{1}{14} \times 10^4 \times 9.8 \times 8 \times 10^{-2} \times 0.3 \times 10^{-3}}{2}$

$T_0 = 0.084$ N/m   For Tube 'B'   नली 'B' के लिए

For Temp 0ºC $\frac{2T_0}{r} = \rho g h_1 \Rightarrow r = \frac{2T_0}{\rho g h_1} = \frac{2 \times 1 \times 0.084}{\frac{1}{14} \times 10^4 \times 9.8 \times 6 \times 10^{-2}}$

0ºC तापमान के लिए $\frac{2T_0}{r} = \rho g h_1 \Rightarrow r = \frac{2T_0}{\rho g h_1} = \frac{2 \times 1 \times 0.084}{\frac{1}{14} \times 10^4 \times 9.8 \times 6 \times 10^{-2}}$

$r = 0.40 \times 10^{-3}$ m $\Rightarrow$ For temp 50ºC $= \frac{2T_{50}}{r} \rho g h$

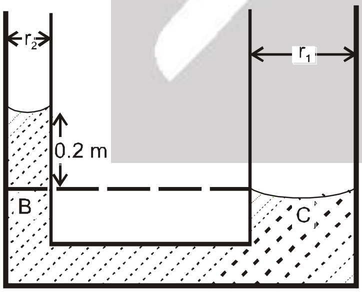
$r = 0.40 \times 10^{-3}$ m $\Rightarrow$ 50ºC तापमान के लिए $\frac{2T_{50}}{r} = \rho g h$

$T_{50} = \frac{\rho g h r}{2} = \frac{\frac{1}{14} \times 10^4 \times 9.8 \times 5.5 \times 10^{-2} \times 0.4 \times 10^{-3}}{2}$

$T_{50} = 0.077 \text{ N/m}^2 \quad = \frac{T_{50} - T_0}{50 - 0} = \frac{0.077 - 0.084}{50} = -1.4 \times 10^{-4}$ N/mºC

Ans. -1.4×10^{-4} N/(m – ºC)

10.



$r_1 = 1.44 \times 10^{-3}$ m. $r_2 = 0.72 \times 10^{-3}$ m.

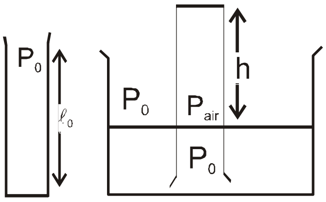
Equating pressures at points (B) & (C) बिन्दु (B) & (C) पर दाब बराबर रखने पर

$P_A - \frac{2\sigma}{r_2} + (0.2)\rho g = P_C$. and तथा $P_B - \frac{2\sigma}{r_1} = P_C$.

so इसलिए $P_B - P_A = 2\sigma \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + 0.2 \rho g = 2 \times 72 \times 10^{-3} \; \frac{N}{m} \left[\frac{10^3}{1.44} - \frac{10^3}{0.72}\right] + (0.2) \times 10^3 \times 9.8$

$= \frac{144 \times (-0.72)}{1.44 \times 0.72} + 1960 = -100 + 1960 = 1860 \text{ N/m}^2.$

11.

Initially 

clearly: $P_{air} - P_0 = \dfrac{2\sigma}{r}$

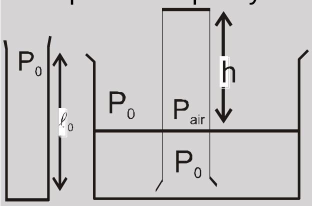
same level has pressure P_0.

and by mole conservation. $\quad P_0 \times \ell_0 = P_{air} \times h \quad$ ...(2)

$= \dfrac{P_0\ell_0}{h} - P_0 = \;;\quad h = \quad \Rightarrow h = \dfrac{P_0 \quad \ell_0}{\dfrac{2\sigma}{r} + P_0}\;$ 0.1 m.

Hence the part inside the water is 0.11 – 0.1 = 0.01 m.

If seal is broken, the atmospheric air will be exerting the pressure which is lesser than P_{air}. Hence the liquid will rise up in the capillary.

हल. प्रारम्भ में 

स्पष्टतः $P_{वायु} - P_0 = \dfrac{2\sigma}{r}$

समान स्तर पर दाब P_0 है।

तथा मोल संरक्षण से $\quad P_0 \times \ell_0 = P_{air} \times h \quad$ ...(2)

$= \dfrac{P_0\ell_0}{h} - P_0 = \dfrac{2\sigma}{r}\;;\quad h = \dfrac{P_0 \quad \ell_0}{\dfrac{2\sigma}{r} + P_0} \Rightarrow h = 0.1$ m.

इस तर पानी के अन्दर का भाग 0.11 – 0.1 = 0.01 m.

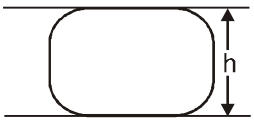
यदि बन्द सिरे को तोड दिया जाये तो वायुमण्डलीय वायु दाब लगायेगी जो कि $P_{वायु}$ से कम होगा। अतः केशनली में द्रव ऊपर उठेगा।

12. Normal rxn at surface $N = \dfrac{2\alpha V\cos\theta}{h^2}$.

सतह पर अभिलम्ब प्रतिक्रिया $N = \dfrac{2\alpha V\cos\theta}{h^2}$.

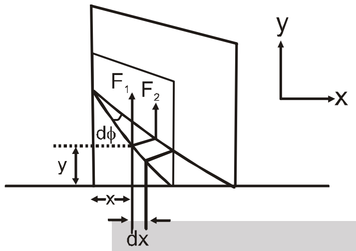
If wetting is complete यदि प्लेटे पानी से पूरी तरह भीगी हुई है $\cos\theta = 1 \Rightarrow N =$

Hence अतः $V = m/p \Rightarrow N = \dfrac{2\alpha m}{\rho h^2}$ Ans.



13. L F_y = (a cos b) dx $\Rightarrow$ F_{net} = 2 Fy = 2a cos b dx. Since
every differential elements are free from each other. Then F_{net} = weight of liquid column of differential length dx $2\alpha \cos \beta$ dx = ρg [yx dϕdx]
प्रत्येक भिन्न–भिन्न अवयव एक दूसरे से स्वतंत्र है, तब F_{net} = dx लम्बाई के द्रव स्तम्भ का भार

$$\Rightarrow y = \frac{(2\alpha \cos \beta o)}{\rho g x d\phi}$$

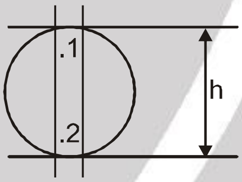


Ans.

14. $R_1 \cong R_2 \cong h/2$. Pressure difference between point (1) and (2): $P_1 = P_0 + \frac{2\alpha}{R_1}$

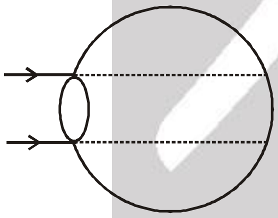
$R_1 \cong R_2 \cong h/2$. बिन्दु (1) तथा (2) के मध्य दाबान्तर : $P_1 = P_0 + \frac{2\alpha}{R_1}$

$$\Rightarrow \quad P_2 = P_0 + \frac{2\alpha}{R_1} \Rightarrow \Delta P = 2\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = h\rho g \Rightarrow 2\alpha\left(\frac{R_1 - R_2}{R_1 R_2}\right) = h\rho g \Rightarrow \frac{2\alpha \Delta R}{\frac{h}{2} \times \frac{h}{2}}$$



= h ρ g $\Rightarrow$ ΔR

15.



We know that force हम जानते है कि बल = $\frac{(dm) \times V}{dt}$

where जहां $\frac{dm}{dt}$ = rate of mass transferred. स्थानान्तरित द्रव्यमान की दर

Now अब $\frac{dm}{dt}$ is also eqval to बराबर है $\frac{dm}{dt} = \frac{\rho \times (\pi b^2) \times (Vdt)}{dt}$ = $\rho\pi b^2$ V.

so force इसलिए बल = $\rho V^2 \pi b^2$

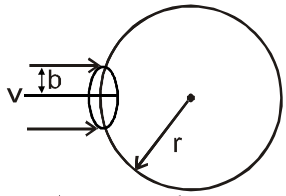
= pressure exerted by air on walls वायु द्वारा दिवारों पर आरोपित दाब = $\frac{\rho V^2 \pi b^2}{\pi b^2}$ = ρV^2.

when the thrust of this pressure becomes equal to the excess pressure
जब यह आधिक्य दाब के बराबर हो जाता है।

$$\Rightarrow \quad \rho V^2 = \frac{4T}{r} \Rightarrow r_{final} = \frac{4T}{\rho V^2}$$

Sol. बुलबुला नलिका से अलग हो जायेगा, यदि हवा के कारण B पर प्रणोद बल, दाब आधिक्य के कारण बल के बराबर होगा।

$\therefore \qquad \rho A v^2 = \left(\frac{4T}{r}\right) A$



(A = B पर बुलबुले का क्षेत्रफल, जहाँ हवा टकराती है)

$\therefore \qquad r = \left(\frac{4T}{\rho v^2}\right)$

Alternate

When force due to surface tension on bubbles is equal to the Force due to blowing air bubble leave contact with ring (separate from ring)

जब पृष्ठ तनाव के कारण बुलबुले पर लगने वाला बल का मान, बहती हुई हवा के कारण बुलबुले पर लगने वाला बल (तब बुलबुला वलय से अलग हो जायेगा।)

$F = 2 \times (2\pi bT) \sin\theta \qquad (\sin\theta = \frac{b}{R})$

$F = 4\pi Tb \left(\frac{b}{R}\right) = \rho\pi b^2 v^2$

$R = \frac{4T}{\rho v^2}$